ROGUE WAVES: A LINEAR APPROXIMATION

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[1] **preliminary report, joint work with Alex Mahalov**.

Goals

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3. Present computer simulations.
4. Discuss some related nonlinear models – second Painlevé transcendents, etc.

Rogue Wave: A Perfect Storm

Rogue Wave: A Moon Night

Ivan Aivazovsky: The Ninth Wave

Rogue Waves: Internet Sources

National Weather Center: official website

http : //www.opc.ncep.noaa.gov/perfectstorm/mpc_ps_rogue.shtml

Wikipedia: article

http : //en.wikipedia.org/wiki/Rogue_wave

YouTube: movie

http : //www.youtube.com/watch?v = yLzgzvVxUV4

Draupner Monster Wave: from BBC documentary

http : //www.youtube.com/watch?v = KJOBOvJEOg

The Deep Blue Monsters: web article

http : //mylifeatsea.blogspot.co.at/2008/01/deep − blue − monsters.html

A Marine Legend: Basic Facts and How to Explain Them?

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3. The freak event that occurred on January 1, 1995 under the **Draupner platform** in the North Sea provides evidence that such waves can occur in the open ocean. During this first scientifically recorded event, an extreme crest with an amplitude of 18.5 m occurred. The maximal wave height of 25.6 m was much more than twice the significant wave height of about 10.8 m.

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3. The freak event that occurred on January 1, 1995 under the **Draupner platform** in the North Sea provides evidence that such waves can occur in the open ocean. During this first scientifically recorded event, an extreme crest with an amplitude of 18.5 m occurred. The maximal wave height of 25.6 m was much more than twice the significant wave height of about 10.8 m.
4. Rogue waves have, over the past twenty or thirty years, come to be recognized as a **unique phenomena** albeit with **several possible causes** *(from The National Weather Center Website)*:

Possible Causes:

1. **Constructive interference:** Several different wave trains of differing speeds and directions meet at the same time. The heights of the crests are additive so that an extreme wave may result when very high waves are included in the wave trains. The effect is normally short lived since the wave trains continue to separate and move on.

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2. **Focusing of wave energy:** When storm forced waves are developed in a water current counter to the wave direction an interaction can take place which results in a shortening of the wave frequency. The result is the superimposing of the wave trains and the generation of extreme waves. Examples of currents where these are sometimes seen are the Gulf Stream and Agulhas current. Extreme wave developed in this regime tend to be longer lived.

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3. **Normal part of the wave spectrum:** The generation of waves on water results not in a single wave height but in a spectrum of waves distributed from the smallest capillary waves to large waves. Within this spectrum there is a finite possibility of each of the wave heights to occur with the largest waves being the least likely. The wave height most commonly observed and forecast is the significant wave height. The random nature of waves implies that individual waves can be substantially higher than the significant wave height. Observations and theory show that the highest individual waves in a typical storm with typical duration to be approximately two times the significant wave height. Waves higher than roughly twice the significant wave height fall into the category of extreme or rogue waves.

Hydrodynamics, Water Waves, and Modulation Instability

▸ Stokes wave

The theory of **deep water waves**, **nonlinear Schrödinger equation**, and **wave statistics**:

- V. E. Zakharov and A. B. Shabat, *Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media*, Zh. Eksp. Teor. Fiz. **61** (1971), 118–134 [Sov. Phys. JETP **34** (1972) #1, 62–69].
- G. B. Whitham, *Linear and Nonlinear Waves*, Wiley-Interscience, New York, 1974.
- R. Smith, *Giant waves*, J. Fluid Mech. **77** (1976) #3, 417–431.
- R. S. Johnson, *On the modulation of water waves in the neighbourhood of* $kh \approx 1.363$, Proc. Royal Soc. London, Ser. A **357** (1977), 131–141.
- H. T. Wist, D. Myrhang, and H. Rue, *Joint distributions of successive wave crest heights and successive wave trouth depths for second-order nonlinear waves*, Journal of Ship Research **46** (2002) # 3, 175–185.
- A. Scott, *Nonlinear Science: Emergence and Dynamics of Coherent Structures*, Second Edition, Oxford University Press, Oxford, 2003.
- A. Scott (Editor), *Encyclopedia of Nonlinear Science*, Routledge, New York and London, 2005.
- F. Fedele, *Extreme events in nonlinear random seas*, Journal of Offshore Mechanics and Arctic Engineering **128** (2006), 11–16.
- V. E. Zakharov and L. A. Ostrovsky, *Modulation instability: The beginning*, Physica D **238** (2009), 540–548.

(See also the numerous references therein.)

Hydrodynamical Models (second Painlevé transcendents)

From **hydrodynamics** to the **nonlinear Schrödinger equations** and giant waves:

- Ronald Smith, *Giant waves*, J. Fluid Mech. **77** (1976) #3, 417–431. pdf
- R. S. Johnson, *On the modulation of water waves in the neighbourhood of* $kh \approx 1.363$, Proc. Royal Soc. London, Ser. A **357** (1977), 131–141. pdf

(Smith) It is suggested that giant waves, as observed on the Agulhas Current, occur where the wave groups are reflected by the current. The local behavior of the wave amplitude is modelled by the nonlinear Schrödinger equation

$$ia_\tau = a_{\rho\rho} - \rho a + \beta |a|^2 a.$$

For waves of a given incident wave amplitude the steady solutions are stable.

1. Introduction During the closure of the Suez Canal a number of ships, particularly oil tankers, have reported extensive damage caused by giant waves off the southeast coast of South Africa (Mallory 1974; Sturm 1974; Sanderson 1974). Two particularly unfortunate vessels are the *World Glory*, which broke in two and sank in June 1968, and the *Neptune Xapphire*, which lost 60 m of its bow section in August 1973. We can only speculate that giant waves may account for many of the ships which have been lost without trace off this coast. When returning from the Persian Gulf the tankers take advantage of the rapid Agulhas Current, and all except one of the eleven incidents listed by Captain Mallory (1974) involved vessels riding on the current. By examining weather charts, Mallory showed that when the incidents occurred the dominant wind-produced waves were opposed by the current...

Hydrodynamical Model (higher nonlinear approximations)

(Johnson) In I967, T. Brooke Benjamin showed that periodic wave-trains on the surface of water could be unstable. If the undisturbed depth is h, and k is the wavenumber of the fundamental, then the Stokes wave is unstable if $kh \geq \sigma_0$, where $\sigma_0 \approx 1.363$. The instability is provided by the growth of waves with a wavenumber close to k. This result is associated with an almost resonant quartet wave interaction and can be obtained by examining the cubic nonlinearity in the nonlinear Schrödinger equation for the modulation of harmonic water waves: **this term vanishes (sic!)** at $kh = \sigma_0$. In this paper the multiple-scales technique is adapted in order to derive the appropriate modulation equation for the amplitude of the fundamental when kh is near to σ_0. The resulting equation takes the form

$$iA_T - a_1 A_{\zeta\zeta} - a_2 A|A|^2 + a_3 A|A|^4 + i(a_4|A|^2 A_\zeta - a_5 A(|A|^2)_\zeta) - a_6 A\psi_T = 0,$$

where $\psi_\zeta = |A|^2$, and the a_i are real numbers. [Coefficients $a_3 - a_6$ are given on $kh \approx 1.363$ only.] This equation is uniformly valid in that it reduces to the **classical non-linear Schrödinger equation** in the appropriate limit and **is correct** when $a_2 = 0$, i. e. at $kh = \sigma_0$.

The equation is used to examine the stability of the Stokes wave and the new inequality for stability is derived: this now depends on the wave amplitude. If the wave is unstable then it is expected that solitons will be produced: the simplest form of soliton is therefore examined by constructing the corresponding ordinary differential equation. Some comments are made concerning the phase-plane of this equation, but more analytical details are extracted by treating the new terms as perturbations of the classical Schrödinger soliton. It is shown that the soliton is both flatter (symmetrically) and skewed forward, although the skewing eventually gives way to an oscillation above the mean level. pdf

Draupner Monster Wave: A Linear Model? Yes! - I think so!

THE SHAPE OF THE DRAUPNER WAVE OF 1^{ST} JANUARY 1995 pdf

P. H. Taylor (paul.taylor@eng.ox.ac.uk)
Department of Engineering Science
University of Oxford

On 1^{st} January 1995 during a relatively severe winter storm at the Draupner platform in the central North Sea, a downwards-pointing laser sensor recorded a time history containing a remarkably large wave. This presentation will discuss the characteristics of this wave with a peak crest elevation of $18.5m$, showing that most of its features can be explained in terms of

1. the average shape of an extreme in a linear random Gaussian process being the scaled auto-correlation function
2. the bound wave structure familiar from Stokes regular wave theory
3. local spectral broadening that occurs when deep water 3^{rd} order wave-wave interactions are important.

However, there are **several curious features** of the large wave **still to be explained**.

Firstly, there is the absence of any local wave group set-down beneath the large crest, instead there is a considerable local set-up. This set-up is a robust feature of the 'freak', distinguishing it all other large (but admittedly not as large) waves in the Draupner records for 1^{st} January.

Secondly, the statistics of the 2^{nd} order sum bound harmonics imply that the non-dimensional water depth is kd$\sim$1.6, **surprisingly close to the critical value of kd=1.36 where the 3^{rd} order wave-wave interactions in the 1-D NLS-equation switch from focusing to de-focusing. (sic!)**

Linear Model: Self-Accelerating Solutions

The time-dependent Schrödinger equation for a free particle

$$i\psi_t + \psi_{xx} = 0, \tag{1}$$

by the following substitution

$$\psi(x,t) = e^{ig\left(x-2gt^2/3\right)t}\, g^{1/3} F\left(g^{1/3}\left(x-gt^2\right)\right), \qquad g = a/2 \tag{2}$$

(a is the acceleration) can be transformed into the Airy equation

$$F'' = zF, \quad z = g^{1/3}\left(x-gt^2\right), \tag{3}$$

whose bounded solutions are the Airy functions $F = k\mathrm{Ai}(z)$ (up to a multiplication constant k) with well-known asymptotics as $z \to \pm\infty$.

Linear Model: Self-Accelerating Solutions (cont.)

Combining with the familiar Galilean transformation,

$$\psi(x,t) = e^{i(x-vt/2)v/2}\,\chi(x-vt,t) \tag{4}$$

(v is the velocity), one obtains a more general solution of this type

$$\begin{aligned}\psi(x,t) &= e^{i(x-vt/2)v/2+ig\left(x-vt-2gt^2/3\right)t} \\ &\quad \times g^{1/3}F\left(g^{1/3}\left(x-vt-gt^2\right)\right).\end{aligned} \tag{5}$$

These freely accelerating Airy beams were theoretically predicted by Berry and Balazs in 1979 in the context of quantum mechanics. They reveal certain remarkable features. The quantity $|\psi(x,t)|^2$ not only remains unchanged in form but also constantly accelerates in empty space. There is no violation of Ehrenfest's theorem because the Airy function is not square integrable. The center of mass of this solution does not exist.

Free Particle: Maximum Kinematical Invariance Group

The largest (known) set of space-time transformations that leaves the system invariant:

$$i\psi_t + \psi_{xx} = 0 \quad \rightarrow \quad i\chi_\tau + \chi_{\xi\xi} = 0, \tag{6}$$

under the following transformation:

$$\begin{aligned}\psi(x,t) &= \frac{1}{\sqrt{\mu(0)(1+4\alpha(0)t)}} \\ &\times \exp i\left(\frac{\alpha(0)x^2+\delta(0)x-\delta^2(0)t}{1+4\alpha(0)t}+\kappa(0)\right) \\ &\times\chi\left(\frac{\beta(0)x-2\beta(0)\delta(0)t}{1+4\alpha(0)t}+\varepsilon(0),\ \frac{\beta^2(0)t}{1+4\alpha(0)t}-\gamma(0)\right).\end{aligned} \tag{7}$$

(It is usually called the Schrödinger group for a free particle.)

U. Niederer, *The maximum kinematical invariance group of the free Schrödinger equation*, Helv. Phy. Acta **45** (1972), 802–810.

U. Niederer, *The maximum kinematical invariance group of the harmonic oscillator*, Helv. Phy. Acta **46** (1973), 191–200.

R. M. López, S. K. Suslov, and J. M. Vega-Guzmán, *Reconstructing the Schrödinger groups*, (to appear in *Physica Scripta*, 12–2012).

Harmonic Oscillator: Maximum Kinematical Invariance Group

- The largest set of space-time transformations that leaves the system invariant.

$$
\begin{array}{ccc}
i\psi_t + \psi_{xx} = x^2\psi & \xrightarrow{\quad S \quad} & i\chi_\tau + \chi_{\xi\xi} = 0 \\
\Big\downarrow T_1 & & \Big\downarrow T_0 \\
i\psi'_{t'} + \psi'_{x'x'} = x'^2\psi' & \xleftarrow{\quad S^{-1} \quad} & i\chi'_{\tau'} + \chi'_{\xi'\xi'} = 0
\end{array}
$$

U. Niederer, *The maximum kinematical invariance group of the Free Schrödinger equation*, Helv. Phy. Acta **45** (1972), 802–810.

U. Niederer, *The maximum kinematical invariance group of the harmonic oscillator*, Helv. Phy. Acta **46** (1973), 191–200.

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Subgroups of the Schrödinger Group

The Schrödinger group includes the familiar *Galilean transformations* given by:

$$\psi(x,t) = \exp\left[i\left(\frac{V}{2}x - \frac{V^2}{4}t\right)\right]\chi(x - Vt + x_0, t - t_0), \tag{8}$$

when $\alpha(0) = \kappa(0) = 0$, $\beta(0) = \mu(0) = 1$, $\gamma(0) = t_0$, $\varepsilon(0) = x_0$ and $\delta(0) = V/2$. These are supplemented by *dilatations,*

$$\psi(x,t) = \chi\left(lx, l^2t\right), \tag{9}$$

where $\alpha(0) = \gamma(0) = \delta(0) = \varepsilon(0) = \kappa(0) = 0$, $\mu(0) = 1$ and $\beta(0) = l$; and *expansions,*

$$\psi(x,t) = \frac{1}{\sqrt{1+mt}}\exp\left(i\frac{mx^2}{4(1+mt)}\right)\chi\left(\frac{x}{1+mt}, \frac{t}{1+mt}\right) \tag{10}$$

$$\left(\mu(0) = 1\,(\neq 0), \qquad \mu'(0) = m\right),$$

$$\psi(x,t) = \frac{1}{\sqrt{2t}}\exp\left(i\frac{x^2}{4t}\right)\chi\left(-\frac{x}{2t}, -\frac{1}{4t}\right) \tag{11}$$

$$\left(\mu(0) = 0, \qquad \mu'(0) = 2\,(\neq 0)\right)$$

with $\beta(0) = 1$, $\delta(0) = \varepsilon(0) = \kappa(0) = 0$. (**Action of any of these transformations on a given solution results in another solution.**)

Linear Case: Self-Acceleration and Self-Compressing Solutions

Freak Wave Solutions: An expansion transformation, namely formula (10) with $m = -1/t_1$, gives the following self-accelerating solution of the free particle equation, $i\psi_t + \psi_{xx} = 0$, in terms of Airy function:

$$\psi(x,t) = \sqrt{\frac{|t_1|}{t_1 - t}} \exp\left(ig\left(x - \frac{2g}{3}\frac{t_1 t^2}{t_1 - t}\right)\frac{t_1^2 t}{(t_1 - t)^2} - \frac{ix^2}{4(t_1 - t)}\right) \times g^{1/3}\mathrm{Ai}\left(g^{1/3}\left(x - g\frac{t_1 t^2}{t_1 - t}\right)\frac{t_1}{t_1 - t}\right), \tag{12}$$

which holds for $t < t_1$. The degenerate case, when $t_1 = 0$, can be analyzed with the help of transformation (11):

$$\psi(x,t) = \frac{1}{\sqrt{2t}} \exp\left(\frac{i}{4t}\left(x^2 + \left(x + \frac{g}{12t}\right)\frac{g}{2t}\right)\right) g^{1/3}\mathrm{Ai}\left(-\frac{g^{1/3}}{2t}\left(x + \frac{g}{8t}\right)\right). \tag{13}$$

From now on, we choose $t_1 > 0$ for the sake of simplicity. It is worth noting that the most general six-parameter solution of this kind can be obtained by formula (7).

A. Mahalov and S. K. Suslov, *An "Airy gun": Self-accelerating solutions of the time-dependent Schrödinger equation in vacuum*, Phys. Lett. A **377** (2012), 33–38. PLA

Finite Energy Airy Beams

Quasi-diffraction free Airy beams are experimentally demonstrated in paraxial optics:

G. A. Siviloglou and D. N. Christodoulides, *Accelerating finite energy Airy beams*, Opt. Lett. **32** (2007) #2, 979–981.

G. A. Siviloglou, J. Broky, A. Dogariu, and D. N. Christodoulides, *Observation of accelerating Airy beams*, Phys. Rev. Lett. **99** (2007), 213901 (4 pages).

Finite Energy Solution: Applying formula (10) with $m = -1/t_1$ to (5), one obtains a more general solution of this kind:

$$\begin{aligned}\psi(x,t) = {} & \sqrt{\frac{|t_1|}{t_1 - t}} \exp\left(i \frac{x - vt/2}{t_1 - t} \frac{vt_1}{2} - \frac{ix^2}{4(t_1 - t)} \right) \\ & \times \exp\left(ig \left(x - vt - \frac{2g}{3} \frac{t_1 t^2}{t_1 - t} \right) \frac{t_1^2 t}{(t_1 - t)^2} \right) \\ & \times g^{1/3} \mathrm{Ai}\left(g^{1/3} \left(x - vt - g \frac{t_1 t^2}{t_1 - t} \right) \frac{t_1}{t_1 - t} \right), \end{aligned} \tag{14}$$

which simplifies to (12) when $v = 0$. The corresponding initial condition is given by

$$\psi(x,0) = e^{-ix^2/4t_1} e^{ivx/2} g^{1/3} \mathrm{Ai}\left(g^{1/3} x \right). \tag{15}$$

Finite Energy Airy Beams (cont.)

By choosing Im v $= -\epsilon$ with $\epsilon > 0$, we arrive at a variant of quasi-diffraction-free finite energy Airy beam which is convenient for experimental observation. Their L^2-norm is **finite**:

$$||\psi||^2 = \int_{-\infty}^{\infty} |\psi(x,t)|^2 \; dx = \int_{-\infty}^{\infty} |\psi(x,0)|^2 \; dx = \sqrt{\frac{g}{4\pi\epsilon}} \exp\left(\frac{\epsilon^3}{12g}\right). \tag{16}$$

Mathematica **allows to derive all (Airy-type) solutions automatically!**

Koutschan's E-Mail: Dec 4, 2012

Mathematica Notebook: deriving Airy solutions automatically

General analytical approach: Proc. Amer. Math. Soc. J. Russian Laser Res.

Koutschan's *Mathematica* Proof: Koutschan.nb

Computer Simulation of a Killer Wave

Features of Killer Waves:

Mallory (1974) describes freak waves as having a steeper forward face preceded by a deep trough, or 'hole in the sea'.

In Bacon's words (1991), these waves 'do not belong to the traditional short term statistical distributions used for ocean waves. The waves are too high, too asymmetric and too steep.'

(Loading movie...)

See *Mathematica* notebook AiryAnimatePLA for more information: nb

Back to Nonlinear Models and Other Applications

1. The study of propagating **nonlinear Airy–Painlevé optical pulses** in dispersive fibers was initiated by Giannini and Joseph in 1989 (see also Smith 1976 for an earlier application of the second Painlevé transcendent in hydrodynamics) and has been continued in recent publications. Similar results hold for a nonlinear parabolic equation in the ionospheric plasma physics.

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2. In the linear case, where self-accelerating Airy beams were first introduced by Berry and Balazs in 1979, we use the symmetry of free Schrödinger equation in order to obtain a more general solution. On the contrary, the $1D$ cubic nonlinear Schrödinger equation is no longer preserved under the expansion transformation. But the same symmetry holds for the quintic nonlinear Schrödinger equation, which is thus invariant under the action of this group of transformations. This is where the **blow up**, namely a singularity such that the wave amplitude tends to infinity in finite time, does exist. Another classical blow up example, where the same symmetry holds, is the $2D$ cubic nonlinear Schrödinger equation (Talanov 1970, Kuznetsov and Turitsyn 1985). The corresponding effects also deserve an experimental study in nonlinear optics.

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3. **Advanced nonlinear models** of rogue waves include the framework of the generalized NL Schrödinger equation accounting for six- and eight-wave interactions, Zakharov's integral equation, two-dimensional models, extensive numerical simulations of wave statistics, stochastic models, etc. (See, for example, a recent preprint ▸ arXiv:1202.5763, articles ▸ PLA and ▸ JOMAE, and the references therein.)

Recent Publications

- R. M. López, S. K. Suslov and J. M. Vega-Guzmán, *On a hidden symmetry of quantum harmonic oscillators*, Journal of Difference Equations and Applications, to appear; see also arXiv:1112.2586v2 [quant-ph] 2 Jan 2012.
- R. M. López, S. K. Suslov, and J. M. Vega-Guzmán, *Reconstructing the Schrödinger groups*, (to appear in *Physica Scripta*, December 2012).
- A. Mahalov and S. K. Suslov, *An "Airy gun": Self-accelerating solutions of the time-dependent Schrödinger equation in vacuum*, Phys. Lett. A **377** (2012), 33–38.
- A. Mahalov and S. K. Suslov, *Wigner function approach to oscillating solutions of the* $1D$*-quintic nonlinear Schrödinger equation*, submitted.
- I. D. Chremmos, Zh. Chen, D. N. Christodoulides, and N. K. Efremidis, *Abruptly autofocusing and autodefocusing beams with arbitrary caustics*, Phys. Rev. A **85** (2012), 023828 (8 pages).
- C. Ament, M. Kolesik, J. V. Moloney, and P. Polynkin, *Self-focusing dynamics of ultraintense accelerating Airy waveforms in water*, Phys. Rev. A **86** (2012), 043842 (6 pages).
- D. S. Agafontsev and V. E. Zakharov, *Rogue waves statistics in the framework of one-dimensional generalized nonlinear Schrödinger equation*, arXiv:1202.5763v3 [physics.optics] 5 Oct 2012.

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Event's website