# ALGORITHMS FOR PARAMETRIZING DEL PEZZO SURFACES OF DEGREE 8 OVER THE RATIONALS

The algorithms for finding a parametrization of Del Pezzo surfaces over $\mathbb{Q}$, if exists, are implemented in Magma, see
`http://magma.maths.usyd.edu.au/`.
We used them with the version V2.12-21.

The package contains the following files:

```
findLieAlg.m
blowup.m
ChevalleyBasis.m
anticanonicalP1xP1.m
csaDeg2.m
ChevalleyBasisQuad.m
anticanonicalSphere.m;
delPezzo8.m
dp8.spec
```

In Magma, load the package by:

```
> AttachSpec("dp8.spec");
```

To compute a parametrization of a Del Pezzo surface you can use the functions

```
ParametrizeDelPezzo8(I)
ParametrizeBlowup(I)
ParametrizeAnticanonicalP1xP1(I)
ParametrizeAnticanonicalSphere(I)
```

As an input they take the vanishing ideal of a particular Del Pezzo surface of degree 8. An ideal in 9 variables generated by quadratic forms is expected. As an output they return a parametrization of the variety as a list of 9 homogeneous polynomials in $s, t, u$ over the rationals describing a rational map from $\mathbb{P}^2$. In case the given variety was not a Del Pezzo surface of degree 8 rational over $\mathbb{Q}$, the empty list is returned.

The first function expects any Del Pezzo surface of degree 8. In case you know which type is your surface of, you can call one of the latter specialized functions. Each one accepts also an optional parameter `statisticsFile`, where a name of the file for more detailed output may be supplied. It will mostly contain a description of the size of data and timings.

There is also a test file in the package which can be loaded by

```
> Attach("Test.m");
```

It contains the functions

```
TestBlowup(max);
```

```
TestAnticanonicalP1xP1(max);
TestAnticanonicalSphere(max, maxf);
```

These functions generate a random example of a particular Del Pezzo surface by perturbing the coefficients of the implicit equations of the standard surface. The argument `max` at the input specifies the maximal absolute value of items in a randomly generated perturbation matrix. The argument `maxf` in the last function the maximum of the absolute value of the discriminant of the splitting field. Again, the name of a file for more detailed output can be supplied via the optional parameter `statisticsFile`.

For sensible values of the arguments `max` and `maxf` (i.e. such that the parametrization of a generated surface is computable in real time), you may consult timings in `http://arxiv.org/abs/math/0512477`.

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