

Theorema 2.0

A Platform for Automated Reasoning in Natural Style

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Theorema: A Brief Description

- Theorema is a **Mathematical Assistant System**.
- Theorema wants to be **more than “just” a theorem prover** (automated, interactive, etc.).
- the **“pencil and paper of the 21st century”**.
- support and facilitate **all aspects** (or at least: **as many as possible**) of the work of a mathematician, e.g.
 - formulation of theorems and entire theories,
 - conjecture new statements,
 - prove conjectured statements,
 - design and implement algorithms,
 - execute algorithms / perform computations,
 - present mathematics in papers / talks / ...
 - ...

What Does Theorema Offer?

- **Powerful internal language:** (Higher order) predicate logic
- **Human-style external syntax** with nice two-dimensional notation for both input and output **AND PROOFS**.
- **Natural Deduction-like** reasoning support for predicate logic (mainly first-order) **PLUS specialized reasoning methods for special areas (e.g. GEOMETRY)**.

(Philosophy: Special reasoning rules are **first** proved correct using basic predicate logic on the object level and **then lifted to the inference level**)

- **Flexible prover-setup** (switchable rules, priorities, etc.).
- **Computation integrated seamlessly** in the basic proving mechanism.
- **Open Source** (available on GitHub), hence open for extensions of any kind!

Short System Demo based on the Formalization of “Interesting Real Mathematics”: AUCTION THEORY

Joint work with M. Kerber, C. Rowat, and C. Lange (University of Birmingham).

From Theoretical Economics: **Second-Price Auctions** (similar to eBay and others)

- auctions of a **single, indivisible good**,
- bidders' private valuations of the auctioned good are **not publicly known**,
- participant submits a **sealed bid**,
- one of the highest bidders wins, and pays the highest **remaining bid**,
(→ “second-price”)
- losers pay nothing.

Vickrey (economics' Nobel prize-winner in 1996) proved:

1. “truth-telling” (bid equal to one's actual valuation) is a **weakly dominant strategy**,
i.e. no bidder can do strictly better by bidding above or below its valuation whatever the other bidders do.
2. auction is **efficient**,
i.e. it allocates the item to the bidder with the highest valuation.

In review (by Eric Maskin) of a highly influential auction theory textbook (by Paul Milgrom), this theorem was listed among the “top-13-theorems” in auction theory.

Why formalization? According to Paul Klemperer (@Oxford): low revenue in some government auctions of 3G mobile communication spectra (20€ vs. 600€ in other countries) is due to **bad auction design**, i.e. the auctions did actually not have the properties that they were intuitively believed to have.

[Auction Theory notebook]

Conclusions & Future Work

- Knowledge archives for distribution of theories (prototype implementation available).
- Exporting theories to MMT/OmDoc format (prototype implementation available).