# The Lagrange Interpolation Formula in Determining the Fluid's Velocity Potential through Profile Grids

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**Abstract.**
Based on the results of [8], [9] and [12], in this paper we present a calculus algorithm for the study of the compressible fluid's stationary movement through profile grids, on an axial–symmetric flow–surface, in variable thickness of stratum. We show the applicability of the boundary element methods (BEM) with real values, and the possibility of solving the integral equation of the velocity potential by using the successive approximation method w.r.t. the parameters $\rho$ (fluid's density) and $h$ (thickness variation of fluid stratum), and using the Lagrangian interpolation formula through five points for the calculation of the derivatives of the velocity potential.

**Keywords:** boundary element method, hydrodynamic networks, Fredholme integral equation, lagrange interpolation

## 1 Presenting the Problem. The Calculus Algorithm for Solving the Integral Equation of Velocity Potential

The fundamental equations (from the CVBEM method) in the problem of the compressible fluid's movement on a axial-symmetric flow–surface, in variable thickness of stratum, could be ( [7], [8], [9]):

$$\begin{aligned} \bar{w}(z) &= \bar{V}_m + \int\limits_{L_0} H(z,\zeta)\bar{w}(\zeta)d\zeta + i\iint\limits_{D^-_{0*}} H(z,\zeta)\widehat{q}(\zeta)d\xi d\eta \\ F(z) &= \bar{V}_m\cdot z + \Gamma\cdot G(z,\zeta_A) + \int\limits_{L_0} H(z,\zeta)F(\zeta)d\zeta + i\iint\limits_{D^-_{0*}} G(z,\zeta)\widehat{q}(\zeta)d\xi d\eta \end{aligned} \tag{1}$$

where:
$A$ – is a fixed point on the base profile $L_0$;
$t$ – is the grid step;
$\Gamma$ – is the circulation around $L_0$.

$$\begin{aligned} &H(z,\zeta) = \tfrac{1}{2ti}\mathrm{ctg}\tfrac{\pi}{t}(z-\zeta) \\ &G(z,\zeta) = \tfrac{1}{2\pi i}\ln\sin\tfrac{\pi}{t}(z-\zeta) \\ &\widehat{q}(\zeta) = 2\tfrac{\partial\bar{w}}{\partial\bar{\zeta}} = -\left[v_x\tfrac{\partial\ln p^*}{\partial\xi} - v_y\tfrac{\partial\ln p^*}{\partial\eta}\right], \qquad p* = \tfrac{\rho\cdot h}{\rho_0} \end{aligned} \tag{2}$$

where:
$\rho$ – is the fluid's density,

$h$ – is a function that represents the thickness' variation of the fluid stratum.

$D_{0*}^{-}$ – bounded simple convex domain, defined as:

$$D_{0*}^{-}: \left[ -\frac{t}{2} < \xi < \frac{t}{2}, -\left(t + \frac{l}{2}\right) < \eta < \left(t + \frac{l}{2}\right) \right] \tag{3}$$

where:

$l$ – is the projection of $L_0$ profile's frame on the $Oy$ axis.

Our purpose is to solve the fundamental equations (1) (obtained from the CVBEM method) using (BEM) in real variables. For doing so, we consider the fundamental integral–equation of the complex potential $F(z) = \varphi + i\psi$ and transform it into an integral equation with real variables, i.e. we build the integral equation of the velocity potential $\varphi(s)$ ($\psi(s)$ is the flow rate function).

**Theorem 1.1.** *[9], [12] In the subsonic motion of the compressible fluid through the profile grid, on an axial–symmetric flow–surface, in variable thickness of stratum, the velocity potential $\varphi(s),\ s \in L_0$ is the solution of the integral equation (4):*

$$\varphi(s) + \int\limits_{L_0} \varphi(\sigma) \frac{dM(s,\sigma)}{d\sigma} d\sigma = b(s) + \iint\limits_{D_{0*}^{-}} \widehat{q}(\sigma) N(s,\sigma) d\xi d\eta \tag{4}$$

*where:*

$s(x_0, y_0)$ *and* $\sigma(\xi, \eta)$ *are the curvilinear coordinates of the fixed point A on the* $L_0$ *base profile;*

$$\begin{aligned}
&b(s) = 2\big(x_0 v_{mx} + y_0 v_{my}\big) + \Gamma M\big(s, \sigma_A\big) + \int\limits_{L_0} \big[\psi(s) - \psi(\sigma)\big] \tfrac{dN}{d\sigma} d\sigma \\
&M(z_0, \zeta) = \tfrac{1}{\pi} arctg \frac{th \frac{\pi}{t}(\eta - y_0)}{tg \frac{\pi}{t}(\xi - x_0)} \\
&N(z_0, \zeta) = \tfrac{1}{\pi} \ln \sqrt{\tfrac{1}{2}\Big[ch \tfrac{2\pi}{t}\big(\eta - y_0\big) - \cos \tfrac{2\pi}{t}\big(\xi - x_0\big)\Big]}
\end{aligned} \tag{5}$$

$v_{mx},\ v_{my}$ *are the components of the asymptotic mean velocity* $v_m$.

**Proposition 1.1.** *[12] In the case of an axial-subsonic movement of a perfect and compressible fluid through profile grids, the flow rate function is determined from the boundary condition (6):*

$$\psi(s) = u_0 \cdot \int_0^s p^*(s) \left(\frac{R}{R_0}\right) ds, \qquad u_0 = \omega R_0, \tag{6}$$

*where:*

- $\omega$ *is the angular rotation velocity of the profile grid;*
- $R_0$ *defines the origin of the axis system related to the turbine's axis.*

## 2 The Successive Approximations Method in Solving the Integral Equation of Velocity Potentials

Equation (4) is an integro–differential equation. In this section, we will show a possibility of solving this equation applying the *method of successive approximation* (the iteration method), using also the result from [10] about the order of the term containing the double integral expression:

$$\varphi_{\widehat{q}}(s) = \iint\limits_{D_{0*}^{-}} \widehat{q}(\sigma) N(s,\sigma) d\xi d\eta. \tag{7}$$

**Proposition 2.1.** *[1] In the case of the subsonic movement of the compressible fluid through the profile grid on an axial–symmetric flow–surface, in variable thickness of stratum, the integral equation of the velocity potential* $\varphi : D_{0*}^{-} \to \Re$ *is solvable by applying the method of successive approximations w.r.t. the parameter* $p^* = \frac{\rho \cdot h}{\rho_0}$.

*Proof.* For isentropic processes , by the Bernoulli–equation, we obtain:

$$\rho = \rho_0 \left(1 - \frac{\gamma - 1}{2} \frac{v^2}{c_0^2}\right)^{\frac{1}{\gamma-1}}, \quad v^2 = v_\tau^2 + v_n^2, \quad v_\tau = \frac{d\varphi}{ds}, \quad v_n = \frac{1}{p^*} \frac{d\psi}{ds} \tag{8}$$

where:

- $\gamma$ is the adiabatic constant;
- $c_0$ is the sound velocity in the zero velocity point;
- $v_\tau$ and $v_n$ are, respectively, the tangential and normal velocities on $L_0$ .

In the first approximation it is assumed that $\rho = \rho_0 =$ constant and $p^* = p^{*(0)} =$ constant. Thus, from (2), it results that $q^{(0)}(\sigma) = 0$. Hence, in the integral equation (4) the double integral (7) is neglected and results the following Fredholme integral equation of second type, with continuous nucleus:

$$\varphi^I(s) + \int\limits_{L_0} \varphi^I(s) \frac{dM(s,\sigma)}{d\sigma} d\sigma = b^I(s) \tag{9}$$

From solving equation (9) we obtain $\varphi^I(s)$, and furthermore from (6), (8), (12) $\psi^I$, $\rho^I$ are obtained. Finally, using the relation:

$$p^* = \frac{\rho \cdot h}{\rho_0}, \qquad \widehat{q}(\sigma) = -\mathrm{grad}\varphi \cdot \mathrm{grad} \ln p^*, \tag{10}$$

a $p^{*I}$ and $\widehat{q}^I(\sigma)$ are determined.

In the second iteration $p^* = p^{*I}$ is assumed and for the determination of $\varphi^{II}(s)$ the following Fredholme integral equation of second type, with continuous nucleus, will be solved:

$$\varphi^{II}(s) + \int\limits_{L_0} \varphi^{II}(s) \frac{dM(s,\sigma)}{d\sigma} d\sigma = b^{II}(s) + \iint\limits_{D_{0*}^{-}} q^I(\sigma) N(s,\sigma) d\xi d\eta, \tag{11}$$

where a $\psi^I$ and $b^{II}(s)$ are previously calculated from (6) and (5), respectively.

From solving equation (11), we obtain $\varphi^{II}$. Furthermore, from (6), (8), (12) and (10) $\psi^{II}$, $\rho^{II}$, $p^{*II}$ and $\widehat{q}^{II}(\sigma)$ are obtained, respectively. Next, the third approximation might be done by assuming $p^* = p^{*II}$, and so on.

□

**Proposition 2.2.** *Having given the values of the velocity potential on each element of the* $L_0$ *profile's division, the tangential velocity* $v_\tau$ *may be calculated in each division element of the* $L_0$ *basic profile's boundary by the formula:*

$$\begin{aligned} &v_{\tau i} = \varphi'(s_i) = \tfrac{2}{3h}(\varphi_{i+2} - \varphi_{i-2}) - \tfrac{1}{12h}(\varphi_{i+4} - \varphi_{i-4}), \\ &h = \Delta s_i = s_{i+1} - s_{i-1}, \\ &i = 1, 3, 5, \ldots, 2n - 1, \end{aligned} \tag{12}$$

*where* $n$ *denotes the number of division elements and by* $s_i$ *we refer to the* $i^{th}$ *element of the division of* $L_0$.

*Proof.* The tangential velocity $v_\tau$ on the $L_0$ basic profile's boundary can be calculated by the well–known formula:

$$v_\tau = \frac{d\varphi}{ds}. \tag{13}$$

Furthermore, as it is also known, the derivative of $\varphi(s_i)$ can be calculated using approximative numerical methods, e.g. using *the interpolation formula of Lagrange through five points* [3] one can calculate the derivative $\varphi'(s_i)$. □

To ensure the practical functionality of proposition 2.1, i.e. to indicate the solving method of the Fredholme integral equation of second type obtained in each approximation step (equation (6), (11) ), let us formulate and prove two more propositions.

**Proposition 2.3.** *[12] In the first approximation step, solving the velocity potential's Fredholme integral equation of second type is reduced to the solving of four systems of linear algebraic equations.*

*Proof.* Using the superposition rule of potential streams, we seek the solution of the Fredholme integral equation of second type (9) to be of the form:

$$\varphi^I = \varphi_1^I v_{mx} + \varphi_2^I v_{my} + \varphi_3^I \Gamma + \varphi_4^I u_0, \qquad u_0 = \omega R_0, \tag{14}$$

where $\varphi_k^I,\ k = 1 \div 4$ are the solutions of the system (15) of integral equations:

$$\begin{aligned}
\varphi_1^I(s) + \int\limits_{L_0} \varphi_1^I(\sigma)\tfrac{dM(s,\sigma)}{d\sigma}d\sigma &= 2x_0 \\
\varphi_2^I(s) + \int\limits_{L_0} \varphi_2^I(\sigma)\tfrac{dM(s,\sigma)}{d\sigma}d\sigma &= 2y_0 \\
\varphi_3^I(s) + \int\limits_{L_0} \varphi_3^I(\sigma)\tfrac{dM(s,\sigma)}{d\sigma}d\sigma &= M(s,\sigma_A) \\
\varphi_4^I(s) + \int\limits_{L_0} \varphi_4^I(\sigma)\tfrac{dM(s,\sigma)}{d\sigma}d\sigma &= b_4(s)
\end{aligned} \tag{15}$$

where:

$$b_4(s) = \int\limits_{L_0} \left[\psi^I(s) - \psi^I(\sigma)\right]\frac{dN}{d\sigma}d\sigma. \tag{16}$$

The integral equations (15) could be solved using the Bogoliubov-Krîlov method, conform to which, solving each integral equation reduces to solving a system of linear algebraic equations. Conform to the method, using an arbitrary division, we partition the boundary of $L_0$ in $n$ subintervals $\Delta s = \Delta\sigma$. Note, that the chosen division might be not uniform, for instance at the trailing or the leading edge, where the variation of the function $\varphi_k^I$ is stronger from point–to–point, the length of subintervals might be shorter. In each subinterval, the function $\varphi_k^I$ is assumed to be constant and equal to $\varphi_{kj}^I$ where $j$ represents the number of the middle–points of the considered subintervals. If the first division–points are debited by even numbers, and the division–points of the middle of the subintervals by odd numbers, then, conform to the approximation method, the integral equations (15) can be approximated by the following systems of linear algebraic equations:

$$\varphi_{ki}^I + \sum_{j=1}^{2n-1} \varphi_{kj}^I \Delta M_{ij} = b_{ki}^I, \qquad i = 1,3,5,\ldots,2n-1, \qquad k = 1,2,3,4, \tag{17}$$

where:

$$
\begin{aligned}
&b_{1i}^I = 2x_i \\
&b_{2i}^I = 2y_i \\
&b_{3i}^I = M_{i,A} \\
&b_{4i}^I = \sum_{j=1}^{2n-1} \Delta\psi_{i,j}^I \left(\frac{dN}{d\sigma}\right)_{i,j} \Delta\sigma_j \\
&\Delta\psi_{i,j}^I = \psi_i^I - \psi_j^I \\
&\Delta\sigma = \sigma_{j+1} - \sigma_{j-1}
\end{aligned} \tag{18}
$$

Solving the algebraic system (17), we obtain $\varphi_{ki}^I$ in $n$ distinct point from the boundary of $L_0$. Finally, from equations (14), $\varphi_i^I$ is determined in each point of the boundary's division. □

**Proposition 2.4.** *[12] In the second approximation step, the Fredholme integral equation (11) of the velocity potential is reduced to solving four systems of linear algebraic equations.*

*Proof.* From (8) and (10), a $\rho^I$ and a $\widehat{q}^I(\sigma)$ is determined, respectively. Consequently, using the superposition rule of potential streams, we seek the solution of the Fredholme integral equation of second type (11) to be of the form:

$$
\varphi^{II} = \varphi_1^{II} v_{mx} + \varphi_2^{II} v_{my} + \varphi_3^{II}\Gamma + \varphi_4^{II} u_0, \qquad u_0 = \omega R_0 \tag{19}
$$

where $\varphi_k^{II},\ k = 1 \div 4$ are the solutions of the system 20 of integral equations:

$$
\begin{aligned}
&\varphi_1^{II}(s) + \int\limits_{L_0} \varphi_1^{II}(\sigma)\frac{dM(s,\sigma)}{d\sigma} d\sigma = 2x_0 + \iint\limits_{D_{0*}^-} \widehat{q}^I(\sigma)N(s,\sigma)d\xi d\eta \\
&\varphi_2^{II}(s) + \int\limits_{L_0} \varphi_2^{II}(\sigma)\frac{dM(s,\sigma)}{d\sigma} d\sigma = 2y_0 + \iint\limits_{D_{0*}^-} \widehat{q}^I(\sigma)N(s,\sigma)d\xi d\eta \\
&\varphi_3^{II}(s) + \int\limits_{L_0} \varphi_3^{II}(\sigma)\frac{dM(s,\sigma)}{d\sigma} d\sigma = M(s,\sigma_A) + \iint\limits_{D_{0*}^-} \widehat{q}^I(\sigma)N(s,\sigma)d\xi d\eta \\
&\varphi_4^{II}(s) + \int\limits_{L_0} \varphi_4^{II}(\sigma)\frac{dM(s,\sigma)}{d\sigma} d\sigma = b_4^{II}
\end{aligned} \tag{20}
$$

where:

$$
\begin{aligned}
&b_4^{II}(s) = \frac{1}{u_0}\int\limits_{L_0} \left[\psi^{II}(s) - \psi^{II}(\sigma)\right]\frac{dN}{d\sigma} d\sigma + \iint\limits_{D_{0*}^-} q^I(\sigma)N(s,\sigma)d\xi d\eta \\
&\psi^{II}(s) = u_0 \int_0^s \left(\frac{R}{R_0}\right)^2 p^{*I}(s)ds \\
&p^{*I} = \frac{\rho^I \cdot h^I}{\rho_0}
\end{aligned} \tag{21}
$$

Using the numeric method presented in proposition 2.3, by applying the Bogoliubov-Krâlov method, solving (20) is reduced to solving systems of linear algebraic equations.

These systems of linear algebraic equations will have the form:

$$
\varphi_{ki}^{II} + \sum_{j=1}^{2n-1} \varphi_{kj}^{II}\Delta M_{ij} = b_{ki}^I, \qquad i = 1,3,5,\ldots,2n-1, \qquad k = 1,2,3,4, \tag{22}
$$

where $b_{1i}^{II}$, $b_{2i}^{II}$, $b_{3i}^{II}$ and $b_{4i}^{II}$ are obtained by using the Simpson formula for handling the double integral.

Solving the algebraic system (22), we obtain $\varphi_{kj}^{II}$ in $n$ distinct point from the boundary of $L_0$. Finally, from equations (19), $\varphi_i^{II}$ ($i = \overline{1,n}$) is determined in each point of the boundary's division.

□

# 3 The Lagrange Interpolation Polynomial. Theory and experiments

The problem of constructing a continuously defined function from given discrete data is unavoidable whenever one wishes to manipulate the data in a way that requires information not included explicitly in the data. The relatively easiest and in many applications often most desired approach to solve the problem is *interpolation* [20], where an approximating function is constructed in such a way as to agree perfectly with the usually unknown original function at the given measurement points. In the practical application of the finite calculus of the problem of interpolation is the following: given the values of the function for a finite set of arguments, to determine the value of the function for some intermediate argument[16].

A chronological overview of the developments in interpolation theory, from the earliest times to the present date could be found in [15]. In this section we focus our attention on the theory of the *lagrange interpolation polynomial* [13], since, as we have already mentioned in the proof of proposition 2.2, its usage arises also in our calculus algorithm for the study of the compressible fluid's stationary movement through profile grids on an axial–symmetric flow–surface in variable thickness of stratum.

## 3.1 The Problem of Interpolation

The problem of interpolation consists in the following[6]: Given the values $y_i$ corresponding to $x_i,\ i = 0, 1, 2, \ldots, n$, a function $f(x)$ of the continuous variable $x$ is to be determined which satisfies the equation:

$$y_i = f(x_i) \text{ for } i = 0, 1, 2 \ldots, n \tag{23}$$

and finally $f(x)$ corresponding to $x = x'$ is required. (i.e. $x'$ different from $x_i, i = \overline{1, n}$.)

In the absence of further knowledge as to the nature of the function this problem is, in the general case, indeterminate, since the values of the arguments other than those given can obviously assigned arbitrarily.

If, however, certain analytic properties of the function be given, it is often possible to assign limits to the error committed in calculating the function from values given for a limited set of arguments. For example, when the function is known to be representable by a polynomial of degree $n$, the value for any argument is completely determinate when the values for $n + 1$ distinct arguments are given.

## 3.2 Lagrange Interpolation

Consider the function $f : [x_0, x_n] \to \mathbb{R}$ given by the following table of values [14]:

| $x_k$ | $x_0$ | $x_1$ | $\ldots$ | $x_n$ |
|---|---|---|---|---|
| $f(x_k)$ | $f(x_0)$ | $f(x_1)$ | $\ldots$ | $f(x_n)$ |

$x_k$ are called *interpolation nodes*, and they are not necessary equally distanced from each other. We seek to find a polynomial $P(x)$ of degree $n$ that approximates the function $f(x)$ in the interpolation nodes, i.e.:

$$f(x_k) = P(x_k);\ k = 0, 1, 2, \ldots, n. \tag{24}$$

The **lagrange interpolation method** finds such a polynomial without solving the system 24.

**Theorem 3.1. Lagrange Interpolating Polynomial**

*The* Lagrange interpolating polynomial *is the polynomial of degree* $n$ *that passes through* $(n+1)$ *points* $y_0 = f(x_0)$, $y_1 = f(x_1), \ldots\ y_n = f(x_n)$ *. It is given by the relation ([21]):*

$$P(x) = \sum_{j=0}^{n} P_j(x) \tag{25}$$

*where:*

$$P_j(x) = y_j \prod_{k=0, k\neq j}^{n} \frac{x - x_k}{x_j - x_k}. \tag{26}$$

*Written explicitly:*

$$\begin{aligned} P(x) = \quad & \tfrac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 + \tfrac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} y_1 + \\ & \cdots + \tfrac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} y_n. \end{aligned} \tag{27}$$

Lagrange interpolating polynomials are implemented in *Mathematica* [22] as `Interpolating Polynomials[data,var]`.

For the case $n = 4$, i.e. interpolation through five points, we have:

$$\begin{aligned} P(x) = \quad & \tfrac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \tfrac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 + \\ & \tfrac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \tfrac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3 + \\ & \tfrac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_2)} y_4 \end{aligned} \tag{28}$$

and

$$\begin{aligned} P'(x) = \quad & \tfrac{(x-x_2)(x-x_3)(x-x_4)+(x-x_1)(x-x_3)(x-x_4)+(x-x_1)(x-x_2)(x-x_4)+(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \\ & \tfrac{(x-x_2)(x-x_3)(x-x_4)+(x-x_0)(x-x_3)(x-x_4)+(x-x_0)(x-x_2)(x-x_4)+(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 + \\ & \tfrac{(x-x_1)(x-x_3)(x-x_4)+(x-x_0)(x-x_3)(x-x_4)+(x-x_0)(x-x_1)(x-x_4)+(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \\ & \tfrac{(x-x_1)(x-x_2)(x-x_4)+(x-x_0)(x-x_2)(x-x_4)+(x-x_0)(x-x_1)(x-x_4)+(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3 + \\ & \tfrac{(x-x_1)(x-x_2)(x-x_3)+(x-x_0)(x-x_2)(x-x_3)+(x-x_0)(x-x_1)(x-x_3)+(x-x_0)(x-x_1)(x-x_2)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_2)} y_4 \end{aligned} \tag{29}$$

Note that the function $P(x)$ passes through the points $(x_i, y_i)$, i.e. $P(x_i) = y_i$.

## 3.3 Applicability of the Lagrange Interpolation for the Study of the Compressible Fluid's Stationary Movement

For illustrating the usability of the Lagrange interpolation method through five points for our calculus algorithm for the study of the compressible fluid's stationary movement through profile grids on an axial–symmetric flow–surface in variable thickness of stratum, namely, for calculating the tangential velocity $v_\tau = \frac{d\varphi}{ds}$ (see section 2, proposition 2.2, equation (13)), consider the following table of values ($i = 0, 1, 2, 3, 4$):

| $s_i$ (in cm) | 10 | 17 | 24 | 31 | 38 |
|---|---|---|---|---|---|
| $\varphi(s_i)$ (in cm/s) | 0.1 | 0.3 | 0.9 | 0.5 | 0.7 |

To obtain the table value representation of $v_\tau = \frac{d\varphi}{ds}$, we use equation (29), hence , for $v_{\tau_0} = \frac{d\varphi}{ds}(s_0)$ we have the relation:

$$\begin{aligned} v_{\tau_0} = \; & \frac{(s_0-s_2)(s_0-s_3)(s_0-s_4)+(s_0-s_1)(s_0-s_3)(s_0-s_4)+(s_0-s_1)(s_0-s_2)(s_0-s_4)+(s_0-s_1)(s_0-s_2)(s_0-s_3)}{(s_0-s_1)(s_0-s_2)(s_0-s_3)(s_0-s_4)}\varphi_0 + \\ & \frac{(s_0-s_2)(s_0-s_3)(s_0-s_4)}{(s_1-s_0)(s_1-s_2)(s_1-s_3)(s_1-s_4)}\varphi_1 + \frac{(s_0-s_1)(s_0-s_3)(s_0-s_4)}{(s_2-s_0)(s_2-s_1)(s_2-s_3)(s_2-s_4)}\varphi_2 + \\ & \frac{(s_0-s_1)(s_0-s_1)(s_0-s_4)}{(s_3-s_0)(s_3-s_1)(s_3-s_2)(s_3-s_4)}\varphi_3 + \frac{(s_0-s_1)(s_0-s_2)(s_0-s_3)}{(s_4-s_0)(s_4-s_1)(s_4-s_2)(s_4-s_2)}\varphi_4 \end{aligned} \tag{30}$$

Using the notation from proposition 2.2, we obtain :

$$v_{\tau_0} = \frac{1}{12h}(-25\varphi_0 + 48\varphi_1 - 36\varphi_2 + 16\varphi_3 - 3\varphi_4). \tag{31}$$

Similarly, we obtain:

$$v_{\tau_1} = \frac{1}{12h}(-3\varphi_0 - 10\varphi_1 + 18\varphi_2 - 6\varphi_3 + \varphi_4) \tag{32}$$

$$v_{\tau_2} = \frac{1}{12h}(\varphi_0 - 8\varphi_1 + 8\varphi_3 - \varphi_4) \tag{33}$$

$$v_{\tau_3} = \frac{1}{12h}(-\varphi_0 + 6\varphi_1 - 18\varphi_2 + 10\varphi_3 + 3\varphi_4) \tag{34}$$

$$v_{\tau_4} = \frac{1}{12h}(3\varphi_0 - 16\varphi_1 + 36\varphi_2 - 48\varphi_3 + 25\varphi_4). \tag{35}$$

Thus, we have the result of proposition 2.2, namely:

$$v_{\tau_2} = \frac{2}{3h}(v_{\tau_3} - v_{\tau_1}) - \frac{1}{12h}(v_{\tau_4} - v_{\tau_0}).$$

Finally, using equations (31)÷(32), we have the table value representation of $v_\tau$, namely:

| $s_i$ (in cm) | 10 | 17 | 24 | 31 | 38 |
|---|---|---|---|---|---|
| $v_\tau(s_i)$ (in cm/s) | $-0.17381$ | 0.12619 | 0.0119 | $-0.08809$ | 0.25476 |

### 3.4 Experimental Results in *Mathematica*

Plotting the given values of $v_\tau$ from section 3.3, yields the following graphic *Mathematica*:

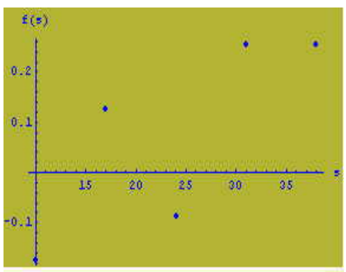


Figure 1: Point Representation of Given Values of $v_\tau$

Applying the lagrangian interpolation method, by the `InterpolatingPolynomial` command of *Mathematica*, the obtained approximation polynomial is:

$$-0.000342119s^4 + 0.003326s^3 - 0.113874s^2 + 1.61754s - 7.94575.$$

Finally, plotting *Mathematica* the obtained approximation polynomial in, we obtain the following graphic, that describes the behavior of $\varphi$:

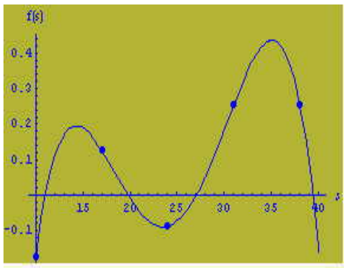


Figure 2: Lagrange Interpolation Polynomial of $v_\tau$

## 4 Conclusion and Further Work

We have shown the usage of the boundary element method with real values for the study of the compressible fluid's stationary movement through profile grids, on an axial–symmetric flow–surface, in variable thickness of stratum. Moreover, we presented a calculus algorithm for solving the integral equation of the velocity potential by using the successive approximation method w.r.t. the parameters $\rho$ (fluid's density) and $h$ (thickness variation of fluid stratum). It turned out, that by using the Lagrangian interpolation formula through five points, the derivatives of the velocity potential can be calculated.

Regarding the Lagrange interpolation method, our plans for the near future are:

- make more test cases w.r.t. several input values of the velocity potentials;
- study the possibility of applying the approximation method for the calculation of other fluid–characteristics.

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