Third training school – RISC 2008

KANT/KASH tutorial

http://www.math.tu-berlin.de/~kant/

LESSENI SYLLA

TU Berlin - Fakultät II

Institut für Mathematik Stra. des

17. Juni 136 D-10623 Berlin, Germany

lesseni(at)math.tu-berlin.de

Plan

Plan

- Introduction to KANT/KASH

Plan

- Introduction to KANT/KASH
- First steps in KASH3

Plan

- Introduction to KANT/KASH
- First steps in KASH3
- Applications

Plan

- Introduction to KANT/KASH
- First steps in KASH3
- Applications
- Programming Language

Introduction to KANT/KASH

- What is KANT/KASH?

Introduction to KANT/KASH

- What is KANT/KASH?
- Starting and Leaving KASH3

Introduction to KANT/KASH

- What is KANT/KASH?
- Starting and Leaving KASH3
- KASH3 - Features

Introduction to KANT/KASH

- What is KANT/KASH?
- Starting and Leaving KASH3
- KASH3 - Features
- KASH3 - Type System

Introduction to KANT/KASH

- What is KANT/KASH?
- Starting and Leaving KASH3
- KASH3 - Features
- KASH3 - Type System
- KASH3 - Help System

Introduction to KANT/KASH

- What is KANT/KASH?
- Starting and Leaving KASH3
- KASH3 - Features
- KASH3 - Type System
- KASH3 - Help System
- KASH3 - Help Examples

Introduction to KANT/KASH

- What is KANT/KASH?
- Starting and Leaving KASH3
- KASH3 - Features
- KASH3 - Type System
- KASH3 - Help System
- KASH3 - Help Examples
- Kant Database

Introduction to KANT/KASH

- What is KANT/KASH?
- Starting and Leaving KASH3
- KASH3 - Features
- KASH3 - Type System
- KASH3 - Help System
- KASH3 - Help Examples
- Kant Database
- QaoS - Future Developments

What is KANT/KASH?

What is KANT/KASH?

- KANT: stands for *Computational Algebraic Number Theory*, with a slight hint at its German origin (Immanuel Kant).
  A sophisticated computer algebra system for computations in algebraic number fields, algebraic function fields and local fields.
  It has been developed under the project leadership of Prof. Dr. M. E. Pohst at the University of Duesseldorf from 1987 until 1993 and at TU Berlin afterwards.
  KANT consists of a C-library of thousands of functions (for doing arithmetic). The set of these functions is based on the core of the computer algebra system MAGMA.

What is KANT/KASH?

The most important database in number theory is available from Berlin (KANT/KASH) by TCP/IP connection and from Bordeaux by ftp connection.

What is KANT/KASH?

- KASH: stands for *KAnt SHell*
  The name of the shell that allows access to the KANT functions. This shell is based on that of the group theory package GAP3 and the handling is similar to that of MAPLE. It makes easier to handle the number theoretical objects.
  The latest version is KASH3 and it is freely available.

The most important database in number theory is available from Berlin (KANT/KASH) by TCP/IP connection and from Bordeaux by ftp connection.

Starting and Leaving KASH3

KASH3 is freely available for
Linux/x86, MacOSX and MS Windows from:
http://www.math.tu-berlin.de/~kant/download.html

Starting and Leaving KASH3

KASH3 is freely available for
Linux/x86, MacOSX and MS Windows from:
http://www.math.tu-berlin.de/~kant/download.html

- Start by simply typing at the prompt: `kash3`

Starting and Leaving KASH3

KASH3 is freely available for
Linux/x86, MacOSX and MS Windows from:
http://www.math.tu-berlin.de/~kant/download.html

- Start by simply typing at the prompt: `kash3`
- Leave by typing at the prompt: `quit;`

NB: the semicolon is necessary!

KASH3 - Features

KASH3 - Features

- Computation in Number Fields: Arithmetic of Algebraic Numbers, Maximal Orders, Integral Bases, Galois Groups up to Degree 23, Unit Groups, Class Groups (unconditionally and under GRH), Class Fields, all Subfields of a Number Field, ...

KASH3 - Features

- Computation in Number Fields: Arithmetic of Algebraic Numbers, Maximal Orders, Integral Bases, Galois Groups up to Degree 23, Unit Groups, Class Groups (unconditionally and under GRH), Class Fields, all Subfields of a Number Field, ...
- Algebraic Function Fields: Arithmetic of Algebraic Functions, Finite and Infinite Maximal Orders, Unit Groups, Riemann Roch Spaces, Divisor Class Groups, ...

KASH3 - Features

- Computation in Number Fields: Arithmetic of Algebraic Numbers, Maximal Orders, Integral Bases, Galois Groups up to Degree 23, Unit Groups, Class Groups (unconditionally and under GRH), Class Fields, all Subfields of a Number Field, ...
- Algebraic Function Fields: Arithmetic of Algebraic Functions, Finite and Infinite Maximal Orders, Unit Groups, Riemann Roch Spaces, Divisor Class Groups, ...
- Local Fields: p-adic Fields, Laurent Series, Polynomial Factorization, ...

KASH3 - Features

- Computation in Number Fields: Arithmetic of Algebraic Numbers, Maximal Orders, Integral Bases, Galois Groups up to Degree 23, Unit Groups, Class Groups (unconditionally and under GRH), Class Fields, all Subfields of a Number Field, ...
- Algebraic Function Fields: Arithmetic of Algebraic Functions, Finite and Infinite Maximal Orders, Unit Groups, Riemann Roch Spaces, Divisor Class Groups, ...
- Local Fields: p-adic Fields, Laurent Series, Polynomial Factorization, ...
- Diophantine Equations: Norm Equations, Thue Equations, Unit Equations, ...

KASH3 - Type System

3 different kinds of Types:

KASH3 - Type System

3 different kinds of Types:

- Simple Types (level 0): stand for themselves

KASH3 - Type System

3 different kinds of Types:

- Simple Types (level 0): stand for themselves
  - Examples:
    `any`, `void`, `char`, `list`, `set`, `string`, `func`

KASH3 - Type System

3 different kinds of Types:

- Simple Types (level 0): stand for themselves
  - Examples:
    `any`, `void`, `char`, `list`, `set`, `string`, `func`
- (Typed) Aggregate Types: are written as

KASH3 - Type System

3 different kinds of Types:

- Simple Types (level 0): stand for themselves
  - Examples:
    `any`, `void`, `char`, `list`, `set`, `string`, `func`
- (Typed) Aggregate Types: are written as
  - `<aggtype>([<type>,<type>])`

KASH3 - Type System

3 different kinds of Types:

- Simple Types (level 0): stand for themselves
  - Examples:
    `any,void,char,list,set,string,func`
- (Typed) Aggregate Types: are written as
  - `<aggtype>([<type>,<type>])`
  - Examples: `map(),seq(),tup()`

KASH3 - Type System

- Composed Types are composed from several atoms and they written as:

KASH3 - Type System

- Composed Types are composed from several atoms and they written as:
  - $[elt-](algebraic\ \ structure)\hat{}(specifier)[/(...)]$

KASH3 - Type System

- Composed Types are composed from several atoms and they written as:
  - $[elt-](algebraic\ \ structure)\hat{}(specifier)[/(...)]$

    atoms for algebraic structure: alg, fld, ord, ...
    atoms for specifier: fun, num, pol, pad, ...

KASH3 - Type System

- Composed Types are composed from several atoms and they written as:
  - $[elt-](algebraic\ \ structure)\hat{}(specifier)[/(...)]$
    atoms for algebraic structure: alg, fld, ord, ...
    atoms for specifier: fun, num, pol, pad, ...
  - Examples:

KASH3 - Type System

- Composed Types are composed from several atoms and they written as:
  - $[elt-](algebraic\ \ structure)\hat{}(specifier)[/(...)]$
    atoms for algebraic structure: alg, fld, ord, ...
    atoms for specifier: fun, num, pol, pad, ...
  - Examples:
    - $fld\hat{}fun$: Type of Function Fields

KASH3 - Type System

- Composed Types are composed from several atoms and they written as:
  - $[elt-](algebraic\ \ structure)\hat{}(specifier)[/(...)]$
    atoms for algebraic structure: alg, fld, ord, ...
    atoms for specifier: fun, num, pol, pad, ...
  - Examples:
    - $fld\hat{}fun$: Type of Function Fields
    - $ord\hat{}num$: Type of Orders of Number Fields

KASH3 - Type System

- Composed Types are composed from several atoms and they written as:
  - $[elt-](algebraic\ \ structure)\hat{}(specifier)[/(...)]$
    atoms for algebraic structure: alg, fld, ord, ...
    atoms for specifier: fun, num, pol, pad, ...
  - Examples:
    - $fld\hat{}fun$: Type of Function Fields
    - $ord\hat{}num$: Type of Orders of Number Fields
    - $elt - ord\hat{}num$: Type of Algebraic Integers

KASH3 - Type System

- Composed Types are composed from several atoms and they written as:
  - $[elt-](algebraic\ \ structure)\hat{}(specifier)[/(...)]$
    atoms for algebraic structure: alg, fld, ord, ...
    atoms for specifier: fun, num, pol, pad, ...
  - Examples:
    - $fld\hat{}fun$: Type of Function Fields
    - $ord\hat{}num$: Type of Orders of Number Fields
    - $elt - ord\hat{}num$: Type of Algebraic Integers
    - $alg\hat{}pol/fld\hat{}num$: Type of Polynomial Algebras over Number Fields

KASH3 - Type System

- Composed Types are composed from several atoms and they written as:
  - $[elt-](algebraic\ \ structure)\hat{}(specifier)[/(...)]$
    atoms for algebraic structure: alg, fld, ord, ...
    atoms for specifier: fun, num, pol, pad, ...
  - Examples:
    - $fld\hat{}fun$: Type of Function Fields
    - $ord\hat{}num$: Type of Orders of Number Fields
    - $elt - ord\hat{}num$: Type of Algebraic Integers
    - $alg\hat{}pol/fld\hat{}num$: Type of Polynomial Algebras over Number Fields
    - $elt - alg\hat{}mat/fld\hat{}rat$: Type of Matrices with Rational Coefficients

KASH3 - Type System

Exercises:

1) The type of complex numbers and the complex field?
2) How to install the new type `elt-case^num`?

KASH3 - Type System

- Useful Functions:

  `ShowTypes();`

  displays a list of all type atoms by level.

  `?*.|TYPE`

  shows all types in KASH3.

  `NewType(newtype,level);`

  installs the type "newtype" of level "level".

Exercises:

1) The type of complex numbers and the complex field?
2) How to install the new type `elt-case^num`?

KASH3 - Help System

For more information, see:
http://www.math.tu-berlin.de/~kant/doc.html

KASH3 - Help System

- On-line documentation

For more information, see:
http://www.math.tu-berlin.de/~kant/doc.html

KASH3 - Help System

- On-line documentation
- Help system written in the shell

For more information, see:
http://www.math.tu-berlin.de/~kant/doc.html

KASH3 - Help System

- On-line documentation
- Help system written in the shell
- Handbooks and tutorials are compiled from documentation records

For more information, see:
http://www.math.tu-berlin.de/~kant/doc.html

KASH3 - Help System

- On-line documentation
- Help system written in the shell
- Handbooks and tutorials are compiled from documentation records
- HTML and print version via XML output

For more information, see:
http://www.math.tu-berlin.de/~kant/doc.html

KASH3 - Help System

- On-line documentation
- Help system written in the shell
- Handbooks and tutorials are compiled from documentation records
- HTML and print version via XML output
- On-line help with complex search patterns

For more information, see:
http://www.math.tu-berlin.de/~kant/doc.html

KASH3 - Help Examples

NB: The kinds of documentation entries are: Keyword, Type, Function, Constant, Statement, Reference, ...

KASH3 - Help Examples

- ? Valuable tool for on-line help

NB: The kinds of documentation entries are: Keyword, Type, Function, Constant, Statement, Reference, ...

KASH3 - Help Examples

- `?` Valuable tool for on-line help
- `?help`

NB: The kinds of documentation entries are: Keyword, Type, Function, Constant, Statement, Reference, ...

KASH3 - Help Examples

- `?` Valuable tool for on-line help
- `?help`
- `?*` Full Text on the types

NB: The kinds of documentation entries are: Keyword, Type, Function, Constant, Statement, Reference, ...

KASH3 - Help Examples

- `?` Valuable tool for on-line help
- `?help`
- `?*` Full Text on the types
- `?name` Description of "name"

NB: The kinds of documentation entries are: Keyword, Type, Function, Constant, Statement, Reference, ...

KASH3 - Help Examples

- `?` Valuable tool for on-line help
- `?help`
- `?*` Full Text on the types
- `?name` Description of "name"
- `?!name` Substrings of "name"

NB: The kinds of documentation entries are: Keyword, Type, Function, Constant, Statement, Reference, ...

KASH3 - Help Examples

- `?` Valuable tool for on-line help
- `?help`
- `?*` Full Text on the types
- `?name` Description of "name"
- `?!name` Substrings of "name"
- `?^name` All functions beginning with "name"

NB: The kinds of documentation entries are: Keyword, Type, Function, Constant, Statement, Reference, ...

KASH3 - Help Examples

- `?` Valuable tool for on-line help
- `?help`
- `?*` Full Text on the types
- `?name` Description of "name"
- `?!name` Substrings of "name"
- `?^name` All functions beginning with "name"
- `?(type)` Give the Input Signature

NB: The kinds of documentation entries are: Keyword, Type, Function, Constant, Statement, Reference, ...

KASH3 - Help Examples

- `?` Valuable tool for on-line help
- `?help`
- `?*` Full Text on the types
- `?name` Description of "name"
- `?!name` Substrings of "name"
- `?^name` All functions beginning with "name"
- `?(type)` Give the Input Signature
- `?->type` Give the Output Types

NB: The kinds of documentation entries are: Keyword, Type, Function, Constant, Statement, Reference, ...

KASH3 - Help Examples

Exercise:
How to get the input signature about number fields, algebraic numbers (elements of number fields) and algebraic integers (elements of orders of number fields) on-line?

Kant Database

QaoS: Query algebraic objects System

http://www.math.tu-berlin.de/~kant/database.html

Kant Database

QaoS: Query algebraic objects System

http://www.math.tu-berlin.de/~kant/database.html

- Database for Field Extensions

contains 1.3 million number fields

Kant Database

QaoS: Query algebraic objects System

http://www.math.tu-berlin.de/~kant/database.html

- Database for Field Extensions
  - Algebraic Extensions

contains 1.3 million number fields

Kant Database

QaoS: Query algebraic objects System

http://www.math.tu-berlin.de/~kant/database.html

- Database for Field Extensions
  - Algebraic Extensions
  - Transcendental Extensions

contains 1.3 million number fields

Kant Database

QaoS: Query algebraic objects System

http://www.math.tu-berlin.de/~kant/database.html

- Database for Field Extensions
  - Algebraic Extensions
  - Transcendental Extensions

  contains 1.3 million number fields
- Database for Transitive groups

  All the 40226 transitive groups of degree up to 30

Kant Database

QaoS: Query algebraic objects System

http://www.math.tu-berlin.de/~kant/database.html

- Database for Field Extensions
  - Algebraic Extensions
  - Transcendental Extensions

  contains 1.3 million number fields
- Database for Transitive groups
  All the 40226 transitive groups of degree up to 30
- Access via HTTP from
  GAP4, KASH2.5, KASH2.6, KASH3, Maple and webbrowser

QaoS - Future Developments

KASH can be downloaded from:
http://www.math.tu-berlin.de/~kant/download.html
Most of the functionality of KASH is also available on the web at:
http://www.math.tu-berlin.de/~kant/calc/

QaoS - Future Developments

- Extension of table of Number Fields

KASH can be downloaded from:
http://www.math.tu-berlin.de/~kant/download.html
Most of the functionality of KASH is also available on the web at:
http://www.math.tu-berlin.de/~kant/calc/

QaoS - Future Developments

- Extension of table of Number Fields
- Tables of Function Fields

KASH can be downloaded from:
http://www.math.tu-berlin.de/~kant/download.html
Most of the functionality of KASH is also available on the web at:
http://www.math.tu-berlin.de/~kant/calc/

QaoS - Future Developments

- Extension of table of Number Fields
- Tables of Function Fields
- Access

KASH can be downloaded from:
http://www.math.tu-berlin.de/~kant/download.html
Most of the functionality of KASH is also available on the web at:
http://www.math.tu-berlin.de/~kant/calc/

QaoS - Future Developments

- Extension of table of Number Fields
- Tables of Function Fields
- Access
  - More clients

KASH can be downloaded from:
http://www.math.tu-berlin.de/~kant/download.html
Most of the functionality of KASH is also available on the web at:
http://www.math.tu-berlin.de/~kant/calc/

QaoS - Future Developments

- Extension of table of Number Fields
- Tables of Function Fields
- Access
  - More clients
  - OpenMath XML data format for transfer of data

KASH can be downloaded from:
http://www.math.tu-berlin.de/~kant/download.html
Most of the functionality of KASH is also available on the web at:
http://www.math.tu-berlin.de/~kant/calc/

QaoS - Future Developments

- Extension of table of Number Fields
- Tables of Function Fields
- Access
  - More clients
  - OpenMath XML data format for transfer of data
  - Bidirectional access will allow external users to add data

KASH can be downloaded from:
http://www.math.tu-berlin.de/~kant/download.html
Most of the functionality of KASH is also available on the web at:
http://www.math.tu-berlin.de/~kant/calc/