Different notions of conuity and intensional models for λ-calculus (Brief Content of the report)

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I. The functional paradigm as a basis for mathematics, logic, and computer science: the role of λ-calculus. The self-applicability principle.

λ-calculus:

Axioms:

$M = M$,
$(\lambda x.M)N = M[x \to N]$,
$\lambda x.M = \lambda y.M[x \to y]$

Rules:

$$\frac{M = N}{N = M}$$

$$\frac{M = N, N = L}{N = L}$$

$$\frac{M = N}{\lambda x.M = \lambda x.N}$$

$$\frac{M = N}{MZ = NZ}$$

$$\frac{M = N}{ZM = ZN}$$

II. Models of λ. D.Scott's topological ("informational") and category-based models. The non-existence of a "naive" model: the Cantor's theorem. Discussion on the groundness axiom. The potential-informative natural interpretation of complete partial order relations between elements of topological models.

Theorem (D.Scott). Given function $f : < A_1, \leq_1 > \to < A_2, \leq_2 >$, where $< A_i, \leq_i >$ are complete partially ordered sets. Then f is Scott-continuous if, and only if, for every directed and non-empty subset X of A_1, the equality $f(\bigvee X) = \bigvee f(X)$ holds.

III. Types (i.e. levels) of abstraction and the concept of positive (i.e. useful) and negative (i.e. useless) information: the pragmatism and goal-driveness. Tasks - as objects, and reductions of descriptable classes of tasks with the smooth abstraction type property - as continuos functions and morphisms.

Let $< A, \leq >$ be any complete and co-complete poset (partially ordered set), P a generalized sequence of its elements (i.e. a sequence with directed set of indexes, not necessarily equal to the set of naturals). Then, in the case if P is increasing (decreasing), we define its limit as the least upper bound (the greatest lower bound) of its elements. In the general case, we put limit of P equal to such an element (if it exists) of A that is the limit of every confinal monotone subsequence of P and the set of all such sequences is non-empty (otherwise, by definition, P is divergent).

For shortness, here we give a new definition of continuity in the case if f is an operation. An operation f on A is said to be $\leq$-continuous if, and only if, it preserves the limits of all the convergent sequences on A.

A ”pictorial” comparison of the new notion of continuity (convergence) with (o)-continuity and Scott's continuity (convergence).

IV. Results

Theorem 1. Let $< A_1, \leq_1 >$ and $< A_2, \leq_2 >$ be complete and co-complete posets, $f : A_1 \to A_2$ be a function. Then f is continuous (in the sense the new notion) if, and only if, the following conditions are satisfied:

(i) if $f(limP)$ is defined then $limf(P)$ is defined, for every sequence P on A_1;

(ii) the restriction of the function f on each set, on which f is monotone, is continuous w.r.t. a corresponding Scott topology.

Theorem 2. It is not true that for every poset $< A, \leq >$, there exists such a topology T, that the set of all $\leq$-continuous operations on A is equal to the set of all the T-continuous operations, and vice versa.

Theorem 3. Investigations of T-convergence and of T-continuity can be formally reduced to investigations of $\leq$-convergence and $\leq$-continuity, and vice versa.