Modified TSR Theory

Khimuri Rukhaia
Jr.Researcher
Institute of Applied Mathematics,TSU
Email: rukhaia@viam.sci.tsu.ge

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Talk outline

Introduction

Language of MτSR theory

Definition of a proof of the theory MτSR

Examples

Language of MτSR theory.

Signs of τSR theory

1)Fundamental sings of τSR theory

1.1. Logical connectives: $\neg$ (of the weight 1), $\vee$, $\leftrightarrow$ (each of the weight 2).

1.2. Logical operational sign τ of the weight (1,1).

1.3. Substantive special substitution operator S of the weight (1,2).

1.4. Relational logico-special substitution operator R of the weight (1,2) and with the logicality indicator 2.

1.5. Object letters:

$$X_0, X_1, X_2, \ldots$$

1.6. Predicate symbols = and $\in$(each of the weight

Predicate letters:

$$P_0^n, Q_0^n, P_1^n, Q_1^n, \ldots \qquad n \in \{0,1,2,\ldots\}$$

and Expressive strings of uppercase letters.

1.7. Functional symbol that has the weight 2, and functional letters:

$$f_0^n, g_0^n, f_1^n, g_1^n, \ldots \quad n \in \{0,1,2,\ldots\}$$

and expressive strings of lowercase letters.

1.8. [and] (left and right brackets).

II. Signs, introduced by the definitions of the types I, II and II‘.

3) Metasigns

3.1. The sybject metacontanty for natural number:

0,1,2,…,10,…

3.2. Metavariables for quantifive sign:

$$x, y, z, x_1, y_1, z_1, \mathrm{K}$$

3.3. Metavariables for forms:

$$\Phi, \Phi_1, \Phi_2, \mathrm{K}$$

3.4. Metavariables for formulae:

$A, B, A_1, B_1, \ldots$

3.5. Metavariables for terms:

$T, U, T_1, U_1, \ldots$

4) Auxilary symbols: **|—, —|, [,] ,**

$\Rightarrow, and, or,$ ***/***, $\cong$, ***MτSR, C, S, D, H, , …,*** Finite sequence of signs of the MJSR theory is called **signcombination**. If signcombination does not contain signs of 4) then we call it word and if the word contains signs from only 1) then it is called **fundamental word**. If the word contains at least one symbol from 2) then we call it **short word.**

Words type τx is logical substantive operators with weight 1 of **MтSR** theory, *Sx* and *Rx* type words are operators with weight 2 of the same theory , besides *Sx* is special substantial partial quantifier with binding indicator 2, *Rx* operator is logico-special relational partial quantifiers with binding and logical indicator 2.

Formulas and terms of τ**SR** theory is defined following way:

1) Object letters, metaconstants for natural numbers, metavariables for object letters and metavariables for terms are atom terms (forms).

2) Metavariables for formulas are atom formulas (forms).

3) Metavariables for forms are atom forms.

4) If σ is n ary logico (special) operator, then $\sigma A_1 \ldots A_n$ is form, i.e is either a

term depending on whether the operator sigma is relational or substantive.

5) If σ is n ary logico-special operator with the logicality indicator $n_1, \ldots, n_k$ and

$\phi_1, \ldots, \phi_n$ are such sequence of forms that

$\phi_{n1}, \ldots, \phi_{n_k}$ is maximal subsequence of formulas from $\phi_1, \ldots, \phi_n$ then $\sigma\phi_1, \ldots, \phi_n$

is a form, i.e is eithe a formula or a term depending on whether the operator σ

is relational or substantive.

6) Words built by only 1-5 are forms(terms and formulas) of **MτSR** theory.

Some **signcombination** we call special prescript signcombination. We separate it when it will be needed. Lets separate some special prescript signcombination:

1. MτSR$| A_1,\ldots,A_n \,|[B_1,\ldots,B_m]A$ is a special prescript signcombination of MτSR theory and reads:

“Theory obtained by extension MτSR *with* $A_1 ,..., A_n$ axioms”

2) MτSR $| A_1,...,A_n | [B_1,...,B_m] A$ is a special prescript signcombination of MτSR theory and reads::” if $B_1,...,B_n$ are theorem s of MτSR $| A_1 ,..., A_n |$theory then A is theorem of MτSR $| A_1,...,A_n |$”

Special prescript signcombination of MτSR $| A_1,...,A_n |$ theory defined by 2) also is called inference rules where $B_1,...,B_n$ formulas are premise and A conclusion of of inference rules.

3) $\phi \cong \phi_1$ is a special prescript signcombination of MτSR theory and reads: " ϕ and ϕ_1 forms are congruent"

4) if E_1 and E_2 are special prescript signcombination of MτSR theory then

$$[E_1 \Rightarrow E_2],[E_1 \wedge E_2],[E_1 \vee E_2]$$

are special prescript signcombination of MτSR theory and reads: "if E_1 then E_2", " E_1 and E_2 ", " E_1 or E_2 " respectively.

5) C0,C1,C2,… are special prescript signcombination of MτSR theory which supports to enumerate inference rules of **MτSR** theory.

Definition of a proof of the theory MτSR.

To finish the construction of the theory **MτSR** it is necessary to introduce the notions denoted by the terms "proof" and "proof text" of the theory **MτSR**. For this we do the following:

1. We first write some formulas of the theory **MτSR**. These formulas are called explicit axioms of the theory **MτSR**, and the letters

occurring in explicit axioms are called bounded constants of the theory **MτSR**.

2. Let us write some character combinations of special purpose in terms of form 2 of the theory **MτSR**. These character combinations are called fundamental derivation rules of the theory **MτSR**.

3. Let is give one or several rules called schemes of the theory **MτSR** .

Any formula formed by applying some schemes of the theory **MτSR** is called an implicit axiom of the theory **MτSR**.

We define the proof of the theory **MτSR** by recursion:

1. Any sequence of explicit and implicit axioms of the theory **MτSR** is a proof of the theory **MτSR**.

2. A sequence $A_1,\ldots, A_m$ of formulas of the theory **MτSR** is a proof of the theory

MτSR if at least one of the following conditions is fulfilled for each formula A of this sequence:

a) a proof, where there occurs *A*, is constructed in **MτSR**;

b) in the sequence mentioned above there exist formulas $A_{i_1},\ldots,A_{i_k},\quad 1\leq i_1<\ldots<i_k\leq m$ that precede *C* and are such that $C_{i_1},\ldots,C_{i_k}$ are the basis of some derivation rule of the theory **MτSR**, while *C* is the derivation of this derivation rule.

A theorem of the theory **MτSR** is a formula occurring in some proof of the theory **MτSR**.

Character combinations of forms

$\vdash$ $M\,\tau SR\ /\ A_1,\ldots,A_m\ /$ (read as “the beginning of the proof in a theory) and

$\dashv$ $M\,\tau SR\ /\ A_1,\ldots,A_m\ /$ (read as “the end of the proof of the theory) $m \geq 0$ are called opposite.

In this case, we call a character combination $\vdash\!\!-\ \mathrm{M}\tau\mathrm{SR}/A_1,\ldots,A_m/$, resp. $-\!\!\mid\ \mathrm{M}\tau\mathrm{SR}/A_1,\ldots,A_m/$, an pening character combination, resp. a closing character combination.

Let us consider the sequence

$$D_1,\ldots,D_n \qquad (1)$$

of closing and opening character combinations.

We say that sequence (1) is normal if the following conditions are fulfilled:

1. D_1 and D_n are the opposite character combinations, D_1 being $\vdash \mathrm{M}\tau\mathrm{SR}$ and D_n being $\dashv \mathrm{M}\tau\mathrm{SR}$

2. For each right character combination D' from sequence (1) there exist left character combinations D'' in (1) corresponding to D' and vice versa. Also, between D' and D'' there occur either only formulas or formulas and only pairs of the corresponding character combinations.

We say that a formula A from (1) is directly connected with the theory $\mathrm{M}\tau\mathrm{SR}/A_1,\ldots,A_m/$ from sequence (1) if $\mathrm{M}\tau\mathrm{SR}/A_1,\ldots,A_m/$ precedes A in (1) and between $\mathrm{M}\tau\mathrm{SR}/A_1,\ldots,A_m/$ and A there may occur formulas and only pairs of the corresponding character combinations. It is obvious that for each formula A of a normal sequence there exists a unique right character combination from this sequence, with which A is connected.

Further, let us consider a subsequence

$$D_1, \ldots, D_i \qquad i = 2, \ldots, m \qquad (2)$$

of sequence (1), where D_i is a formula A. From (2) we first remove all terms occurring between the pairs of the corresponding character combinations and then all terms which are not a formula. We call the remainder a subsequence of (1) connected with A.

We call the normal sequence $D_1, \dots, D_n$ the conclusive text of the theory MτSR if for each formula *A* from (1) connected with $\vdash$MτSR/ $A_1, \dots, A_m$/ in (1) the following condition is fulfilled:

1. *A* subsequence of sequence (1) connected with A is a proof of the theory MτSR / $A_1, \dots, A_m$ /

The character combination ***A..***, resp. *A.Cm*

,

resp. $A.Sm$, resp. $A.SDm$ read as "A is a theorem according to the conditions, resp. Cm , resp. Sm , resp. $A.SDm$". In these cases we say that a formula A is given with commentary.

The axiom schemes of the theory and the inference rules are the following:

$$S1. \quad \to \vee AAA$$

$$S2. \to \; A \vee \; AB$$

$$S3. \to \; \vee \; AB \quad \vee \; BA$$

$$S4. \to \to AB \to \to BA_1 \to AA_1$$

$$S5. \to \to \; AB \; \to \vee A_1 A \vee \; A_1 B$$

$$S6. \to \leftrightarrow \quad AB \quad \leftrightarrow \quad BA$$

$$S7. \to \leftrightarrow \quad AB \quad \to \quad AB$$

$$S8. \to \quad RxTA \quad \exists xA$$

$$S9. \leftrightarrow \; RxTA \quad (T \; / \; x) \, A$$

a. if A and $\vee\neg AB$, then B;

b. if A and $\leftrightarrow AB$, then B;

c. if $\leftrightarrow AB$, then $\leftrightarrow BA$;

d. if A is congruence of B, then B;

HAD_m. Last,assume $C-C_1$ is a D_m $(m=1,2,...)$ definition, then $C\leftrightarrow C_1$ (respectively $C=C_1$) is an axiom schema if *C* is a formula (eresp. *C i*s a term).

$$C0. \quad \mathrm{M}\,\tau\mathrm{SR}\,[A](T/X)A$$

$$C1. \quad \mathrm{M}\,\tau\mathrm{SR}[A, \leftrightarrow AB]B$$

$$C2. \quad \mathrm{M}\tau\mathrm{SR}[A, \leftrightarrow BA]B$$

$$C3. \quad \mathrm{M}\,\tau\mathrm{SR}\left|A\right|B \Rightarrow \mathrm{M}\,\tau\mathrm{SR} \rightarrow AB$$

$$C5. \quad \mathrm{M}\,\tau\mathrm{SR}[A, \rightarrow AB]B$$

$$C11. \quad \mathrm{M}\,\tau\mathrm{SR} \vee A \neg A$$

/begin proof of $\mathrm{M}\ \tau \mathrm{SR}$ **/**

/ begin proof of the theory extended by $\neg A$ **axiom/**

/B is a theorem by condition/

/ $\rightarrow \neg B \vee \neg BA$ **is a theorem by S2/**

/$\vee\ \neg\ BA$ **is a theorem by C5/**

/*A* is a theorem by C1 /

/finished proof of the theory extended by axiom $\neg\ A$ **/**

/ $\to\neg AA$ **is a theorem by C3/**

/ $\to\neg AA \to\vee A\neg A\vee AA$ **is a theorem by S5/**

/$\to\vee A\neg A\vee AA$ **is a theorem by C5/**

/ $\vee\ A\neg A$ **is a theorem by C11/**

/ $\vee AA$ **is a theorem by C5/**

/$\to\vee AAA$ **is a theorem by S1/**

/A **is a theorem by C5/**

/finished proof in $\mathrm{M}\tau\mathrm{SR}$ **/**

$C15. \quad M\tau SR[\neg A]B \text{ and } M\tau SR[\neg A]\neg B \Rightarrow M\tau SRA.$

$\vdash, M\ \tau SR$

$\vdash, M\ \tau SR\ [\neg A]$

$B ..,$

$\neg B ..,$

$\rightarrow \neg B \vee \neg BA. \quad S2,$

$\vee \neg BA. \quad C5,$

$A. \quad C1,$

$\dashv\ M\ \tau SR\ [\neg A]$

.

$\rightarrow \neg AA. \quad C3,$

$\rightarrow \neg AA \rightarrow \vee A \neg A \vee AA. \quad S5,$

$\rightarrow \vee A \neg A \vee AA. \quad C5,$

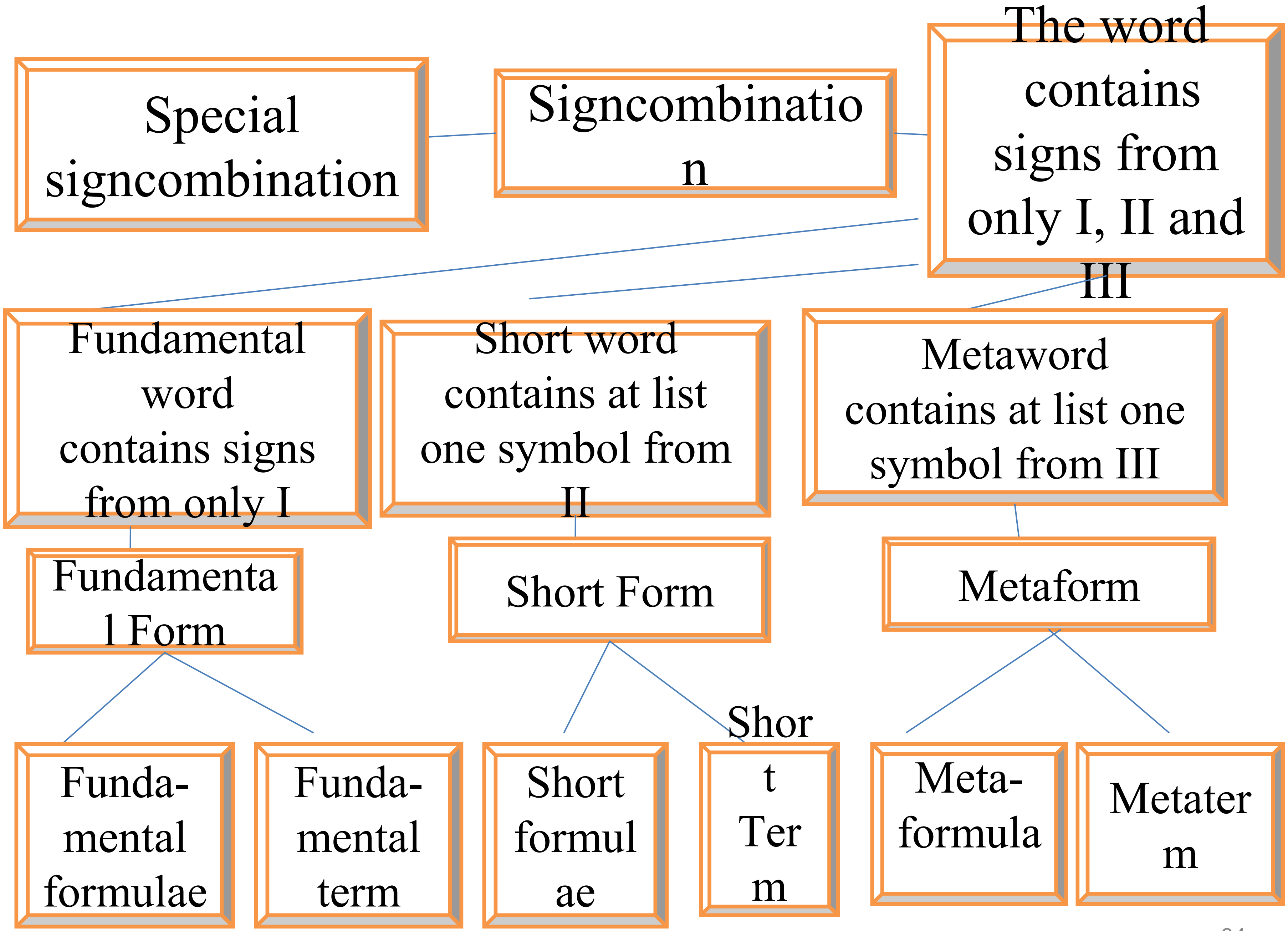
$\vee A \neg A. \quad C11,$

$\vee AA. \quad C5,$

$\rightarrow \vee AAA. \quad S1,$

$A. \quad C5,$

$\dashv M \tau SR$


Special signcombination
Signcombination
The word contains signs from only I, II and III
Fundamental word contains signs from only I
Short word contains at list one symbol from II
Metaword contains at list one symbol from III
Fundamental Form
Short Form
Metaform
Funda-mental formulae
Funda-mental term
Short formulae
Short Term
Meta-formula
Metaterm

The logical form of the recursive definition

1:ancestor$(x,y) \leftrightarrow$ parent(x,y)

N:ancestor$(x,y) \leftrightarrow \exists z$ (parent$(x,z) \wedge$ N-1:ancestor (z,y)).

N+M:ancestor$(x,y) \leftrightarrow$

$\exists z$ (N:ancestor$(x,z) \wedge$ M:ancestor(z,y))