We have to prove:

$((A \Rightarrow C) \wedge (B \Rightarrow A)) \Leftrightarrow ((A \vee B) \Rightarrow C)$ (Inv)

with no assumptions.

Proof by sequent calculus with unit propagation:

{{((A ⇒ C) ∧ (B ⇒ A)) ⇔ ((A ∨ B) ⇒ C), "Inv"}}. (G#0)

Equivalence goal (Inv) is proven in both directions.

☑ {1} Direction from left to right: Assume:

$(A \Rightarrow C) \wedge (B \Rightarrow A)$, (A#1)

and prove:

$(A \vee B) \Rightarrow C$, (G#2)

{{(A ∨ B) ⇒ C, "G#2"}}. (G#3)

Assumed conjunction (A#1) is split into:

$A \Rightarrow C$, (A#1.1)

$B \Rightarrow A$. (A#1.2)

Implicative goal (G#2) is split. Assume:

$A \vee B$, (A#17)

and prove:

C, (G#18)

{{C, "G#18"}}. (G#19)

Disjunctive assumption (A#17) is used for proof by cases.

☑ {1, 1} Case (A#17.1):

A. (A#17.1)

Implicative assumption (A#1.1) is used for Modus Ponens.

☑ {1, 1, 1} First the premise of (A#1.1) is proven:

A. (G#38)

{{A, "G#38"}, {C, "G#18"}}. (G#39)

Assumed (A#17.1) goal (G#38): success.

☑ {1, 1, 2} Now the conclusion of (A#1.1) is assumed:

C. (A#37)

Assumed (A#37) goal (G#18): success.

☑ {1, 2} Case (A#17.2):

B. (A#17.2)

Implicative assumption (A#1.1) is used for Modus Ponens.

☑ {1, 2, 1} First the premise of (A#1.1) is proven:

```
A.                                                    (G#41)
{{A, "G#41"}, {C, "G#18"}}.                           (G#42)
```

Implicative assumption (A#1.2) is used for Modus Ponens.

☑ {1, 2, 1, 1} First the premise of (A#1.2) is proven:

```
B.                                                    (G#44)
{{B, "G#44"}, {A, "G#41"}, {C, "G#18"}}.              (G#45)
```

Assumed (A#17.2) goal (G#44): success.

☑ {1, 2, 1, 2} Now the conclusion of (A#1.2) is assumed:

```
A.                                                    (A#43)
```

Assumed (A#43) goal (G#41): success.

☑ {1, 2, 2} Now the conclusion of (A#1.1) is assumed:

```
C.                                                    (A#40)
```

Assumed (A#40) goal (G#18): success.

⚠ {2} Direction from right to left: Assume:

```
(A ∨ B) ⇒ C,                                          (A#4)
```

and prove:

```
(A ⇒ C) ∧ (B ⇒ A),                                    (G#5)
{{(A ⇒ C) ∧ (B ⇒ A), "G#5"}}.                         (G#6)
```

Conjunctive goal (G#5) is decomposed on different proof branches.

☑ {2, 1} Proof of (G#5.1):

```
A ⇒ C.                                                (G#5.1)
{{A ⇒ C, "G#5.1"}}.                                   (G#46)
```

Implicative goal (G#5.1) is split. Assume:

```
A,                                                    (A#52)
```

and prove:

```
C,                                                    (G#53)
{{C, "G#53"}}.                                        (G#54)
```

Implicative assumption (A#4) is used for Modus Ponens.

☑ {2, 1, 1} First the premise of (A#4) is proven:

```
A ∨ B.                                                (G#62)
{{A ∨ B, "G#62"}, {C, "G#53"}}.                       (G#63)
```

Disjunctive goal (G#62) is split:

```
A,                                                    (G#62.1)
B.                                                    (G#62.2)
```

```
{{A, "G#62.1"}, {B, "G#62.2"}, {C, "G#53"}}.
```
(G#64)

Assumed (A#52) goal (G#62.1): success.

☑ {2, 1, 2} Now the conclusion of (A#4) is assumed:

```
C.
```
(A#61)

Assumed (A#61) goal (G#53): success.

⚠ {2, 2} Proof of (G#5.2):

```
B ⇒ A.
```
(G#5.2)

```
{{B ⇒ A, "G#5.2"}}.
```
(G#47)

Implicative goal (G#5.2) is split. Assume:

```
B,
```
(A#65)

and prove:

```
A,
```
(G#66)

```
{{A, "G#66"}}.
```
(G#67)

Implicative assumption (A#4) is used for Modus Ponens.

☑ {2, 2, 1} First the premise of (A#4) is proven:

```
A ∨ B.
```
(G#75)

```
{{A ∨ B, "G#75"}, {A, "G#66"}}.
```
(G#76)

Disjunctive goal (G#75) is split:

```
A,
```
(G#75.1)

```
B.
```
(G#75.2)

```
{{A, "G#75.1"}, {B, "G#75.2"}, {A, "G#66"}}.
```
(G#79)

Assumed (A#65) goal (G#75.2): success.

⚠ {2, 2, 2} Now the conclusion of (A#4) is assumed:

```
C.
```
(A#74)

Proof by UP fails (no applicable rule).

Goal:

```
{{A, "G#66"}}.
```

Assumptions:

```
B,
```
(A#65)

```
C.
```
(A#74)