

We prove:

$$(A \wedge B) \Rightarrow C \Leftrightarrow ((A \Rightarrow C) \vee (B \Rightarrow C)) \quad (\text{abc2})$$

with no assumptions.

Proof by sequent calculus with unit propagation:

$$\{ \{ (A \wedge B) \Rightarrow C \Leftrightarrow ((A \Rightarrow C) \vee (B \Rightarrow C)), "abc2" \} \}. \quad (\text{G\#0})$$

Equivalence goal (abc2) is proven in both directions.

☑ {1} Direction from left to right: Assume:

$$(A \wedge B) \Rightarrow C, \quad (\text{A\#1})$$

and prove:

$$(A \Rightarrow C) \vee (B \Rightarrow C), \quad (\text{G\#2})$$

$$\{ \{ (A \Rightarrow C) \vee (B \Rightarrow C), "G\#2" \} \}. \quad (\text{G\#3})$$

Disjunctive goal (G#2) is split:

$$A \Rightarrow C, \quad (\text{G\#2.1})$$

$$B \Rightarrow C. \quad (\text{G\#2.2})$$

$$\{ \{ A \Rightarrow C, "G\#2.1" \}, \{ B \Rightarrow C, "G\#2.2" \} \}. \quad (\text{G\#9})$$

Implicative goal (G#2.1) is split. Assume:

$$A, \quad (\text{A\#14})$$

and prove:

$$C, \quad (\text{G\#15})$$

$$\{ \{ C, "G\#15" \}, \{ B \Rightarrow C, "G\#2.2" \} \}. \quad (\text{G\#16})$$

Implicative goal (G#2.2) is split. Assume:

$$B, \quad (\text{A\#23})$$

and prove:

$$C, \quad (\text{G\#24})$$

$$\{ \{ C, "G\#15" \}, \{ C, "G\#24" \} \}. \quad (\text{G\#25})$$

Implicative assumption (A#1) is used for Modus Ponens.

☑ {1, 1} First the premise of (A#1) is proven:

$$A \wedge B. \quad (\text{G\#33})$$

$$\{ \{ A \wedge B, "G\#33" \}, \{ C, "G\#15" \}, \{ C, "G\#24" \} \}. \quad (\text{G\#34})$$

Conjunctive goal (G#33) is decomposed on different proof branches.

☑ {1, 1, 1} Proof of (G#33.1):

$$A. \quad (\text{G\#33.1})$$

$$\{ \{ A, "G\#33.1" \}, \{ C, "G\#15" \}, \{ C, "G\#24" \} \}. \quad (\text{G\#35})$$

Assumed (A#14) goal (G#33.1): success.

☑ {1, 1, 2} Proof of (G#33.2):

B.

(G#33.2)

{B, "G#33.2"}, {C, "G#15"}, {C, "G#24"}.

(G#36)

Assumed (A#23) goal (G#33.2): success.

☑ {1, 2} Now the conclusion of (A#1) is assumed:

C.

(A#32)

Assumed (A#32) goal (G#15): success.

☑ {2} Direction from right to left: Assume:

$(A \Rightarrow C) \vee (B \Rightarrow C)$,

(A#4)

and prove:

$(A \wedge B) \Rightarrow C$,

(G#5)

{(A ∧ B) ⇒ C, "G#5"}.

(G#6)

Implicative goal (G#5) is split. Assume:

A ∧ B,

(A#37)

and prove:

C,

(G#38)

{C, "G#38"}.

(G#39)

Assumed conjunction (A#37) is split into:

A,

(A#37.1)

B.

(A#37.2)

Disjunctive assumption (A#4) is used for proof by cases.

☑ {2, 1} Case (A#4.1):

$A \Rightarrow C$.

(A#4.1)

Implicative assumption (A#4.1) is used for Modus Ponens.

☑ {2, 1, 1} First the premise of (A#4.1) is proven:

A.

(G#60)

{A, "G#60"}, {C, "G#38"}.

(G#61)

Assumed (A#37.1) goal (G#60): success.

☑ {2, 1, 2} Now the conclusion of (A#4.1) is assumed:

C.

(A#59)

Assumed (A#59) goal (G#38): success.

☑ {2, 2} Case (A#4.2):

$B \Rightarrow C$.

(A#4.2)

Implicative assumption (A#4.2) is used for Modus Ponens.

☑ {2, 2, 1} First the premise of (A#4.2) is proven:

B.

(G#63)

{ {B, "G#63"}, {C, "G#38"} }.

(G#64)

Assumed (A#37.2) goal (G#63): success.

☑ {2, 2, 2} Now the conclusion of (A#4.2) is assumed:

C.

(A#62)

Assumed (A#62) goal (G#38): success.