We prove:

```
((A ∨ B) ⇒ C) ⇔ ((A ⇒ C) ∧ (B ⇒ C))                    (abc1)
```

with no assumptions.

Proof by sequent calculus with unit propagation:

```
{{((A ∨ B) ⇒ C) ⇔ ((A ⇒ C) ∧ (B ⇒ C)), "abc1"}}.       (G#0)
```

Equivalence goal (abc1) is proven in both directions.

☑ {1} Direction from left to right: Assume:

```
(A ∨ B) ⇒ C,                                           (A#1)
```

and prove:

```
(A ⇒ C) ∧ (B ⇒ C),                                     (G#2)
{{(A ⇒ C) ∧ (B ⇒ C), "G#2"}}.                          (G#3)
```

Conjunctive goal (G#2) is decomposed on different proof branches.

☑ {1, 1} Proof of (G#2.1):

```
A ⇒ C.                                                 (G#2.1)
{{A ⇒ C, "G#2.1"}}.                                    (G#9)
```

Implicative goal (G#2.1) is split. Assume:

```
A,                                                     (A#15)
```

and prove:

```
C,                                                     (G#16)
{{C, "G#16"}}.                                         (G#17)
```

Literal assumption (A#15) is propagated !

Using literal assumption (A#15), assumption (A#1) is simplified to:

```
C.                                                     (A#24)
```

Assumed (A#24) goal (G#16): success.

☑ {1, 2} Proof of (G#2.2):

```
B ⇒ C.                                                 (G#2.2)
{{B ⇒ C, "G#2.2"}}.                                    (G#10)
```

Implicative goal (G#2.2) is split. Assume:

```
B,                                                     (A#33)
```

and prove:

```
C,                                                     (G#34)
{{C, "G#34"}}.                                         (G#35)
```

Literal assumption (A#33) is propagated !

Using literal assumption (A#33), assumption (A#1) is simplified to:

C. (A#42)

Assumed (A#42) goal (G#34): success.

☑ {2} Direction from right to left: Assume:

(A ⇒ C) ∧ (B ⇒ C), (A#4)

and prove:

(A ∨ B) ⇒ C, (G#5)

{{(A ∨ B) ⇒ C, "G#5"}}. (G#6)

Assumed conjunction (A#4) is split into:

A ⇒ C, (A#4.1)

B ⇒ C. (A#4.2)

Implicative goal (G#5) is split. Assume:

A ∨ B, (A#59)

and prove:

C, (G#60)

{{C, "G#60"}}. (G#61)

Literal goal (G#60) is propagated !

Using literal goal (G#60), assumptions (A#4.1) and (A#4.2) are simplified to:

¬ A, (A#70)

¬ B. (A#71)

{}. (G#72)

Literal assumption (A#70) is propagated !

Using literal assumption (A#70), assumption (A#59) is simplified to:

B. (A#87)

Contradictory assumptions (A#87), (A#71): success.