

First Order Predicate Logic

6. Equality

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Predicate Logic with Equality

$=$ is a "standard" predicate in the basic syntax
and has a "standard" semantics:

In every interpretation I ,

$=_I$ is the diagonal relation on D_I : $\{\langle d, d \rangle \mid d \in D_I\}$.

Substitution

Instantiate to ground formula:

$\forall_{x,y} \dots (\varphi \Rightarrow t_1 = t_2)$

Example: $\forall_{x,y} ((x > y) \Rightarrow G[x, y] = G[y, r[x, y]])$

$(15 > 10) \Rightarrow G[15, 10] = G[10, r[15, 10]] \longrightarrow G[15, 10] = G[10, 5]$

Replacement

Use ground equality $t_1 = t_2$ to transform ground term t into t' by
replacing one occurrence of t_1 in t by t_2 .

Example: use $G[15, 10] = G[10, 5]$ to transform $G[15, 10]$ into $G[10, 5]$

Computation is a sequence of substitutions and replacements: a proof
that the initial value equals the result, in which every step is "easy" to
decide. (See also [reverse.pdf](#) from the first lecture.)