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Ordinary Differential Equations and Dynamical Systems – Exam 28.1.2014

1. Find the general solution of each of the following equations.
a) $y'(x)\sin(y) - \sin(x) = 0$.
b) $y'(x) - y(x) = e^{2x}$.

2. Consider the following variational problem: for $a, b \in \mathbb{R}$, find $f : [a, b] \to \mathbb{R}$ subject to $f(a) = f(b) = 0$ optimizing the value of the integral

$$\int_a^b (f'(x)^2 + xf(x))dx.$$

a) Give the Euler-Lagrange equation.
b) Give an equivalent System of first order differential equations.

3. Find the equilibria in the vector field

$$F : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (x, \sin(y))$$

and check whether they are hyperbolic, and if yes, if they are sources or sinks or saddles.

Bringing your lecture notes/books is permitted. Using electronic devices is not allowed.