Fundamentals Num. Analysis & Symb. Comp.
Homework Schicho

In the exercises below, the participant is asked to prove some theorems in kinematics. Both theorems are known, but proofs without bond theory would require complicated computation. The proofs below should be done without computer computations, using only facts and concepts that have been treated in the lecture on April 7.

1 Mobile PRRRR Linkages

The closure equation of a PRRRR linkage can be written as the equations in $(t_1, \ldots, t_5)$

$$(t_5 - \epsilon v)(t_1 - h_1)(t_2 - h_2)(t_3 - h_3)(t_4 - h_4) \sim 1,$$

with parameters $v \in \mathbb{H}$ purely vectorial (i.e. $\bar{v} = -v$) specifying the translation direction of the first joint and h_1, h_2, h_3, h_4 such that $h_i^2 = -1$ for $i = 1, \ldots, 4$ specifying the rotation axes in some initial position.
Assume that the PRRRR linkage is mobile, without frozen joint and coinciding axes. Show that either $\epsilon h_1 = \pm\epsilon h_2$ or $\epsilon h_3 = \pm\epsilon h_4$.

Remark. The condition $\epsilon h_1 = \pm\epsilon h_2$ is equivalent to the statement that the two axes are parallel.

2 Mobile SRRR Linkages

The closure equation of an SRRR linkage can be written as the equations
in $(t_1, t_2, t_3, q) \in (\mathbb{R} \cup \{\infty\})^3 \times (\mathbb{H} \setminus \{0\})$

$$(t_1 - h_1)(t_2 - h_2)(t_3 - h_3)q \sim 1,$$

with parameters h_1, h_2, h_3 such that $h_i^2 = -1$ for $i = 1, 2, 3$ specifying the rotation axes in some initial position.
Assume that the axes h_1 and h_3 do not pass through the origin (the center of the S-joint), which is equivalent to saying $h_1, h_3 \notin \mathbb{H}$. Assume that the SRRR linkage is mobile, without frozen joint and coinciding axes. Show that the three rotation axes form a bonded chain.

Remark. By a theorem from the lecture, three lines form a bonded chain if and only if there exists a fourth line such that the four lines form a mobile 4R loop. This is known as Bennett's condition; it is a codimension 2 condition.

Remark 2. Bonds for spherical joints are defined as solutions of the equation above with $t_1, t_2, t_3 \in \mathbb{C}$ and $q \in \mathbb{DHC} \setminus \{0\}$, where $\mathbb{DHC} = \mathbb{DH} \oplus i\mathbb{DH}$.