EXERCISES-10

(1) Let $G = S_3$ and $X = \{0,1\}^{\binom{[3]}{2}} = \left\{ f \mid f : \binom{[3]}{2} \to \{0,1\} \right\}$. Define $\phi : G \times X \to X$ by $\phi(\sigma, f) := g$ with $g(\sigma\{x,y\}) := f(\{x,y\})$, or equivalently, $g(\{x,y\}) := f(\{\sigma^{-1}(x), \sigma^{-1}(y)\})$. Here $\sigma\{x,y\} = \{\sigma(x), \sigma(y)\}$. Let f_1 be an element of X given by

$$f_1 := \begin{pmatrix} \{1,2\} & \{1,3\} & \{2,3\} \\ 1 & 0 & 1 \end{pmatrix}.$$

Compute the orbit of f_1; i.e., compute

$$S_3 f_1 = \{\phi(\sigma, f_1) \mid \sigma \in S_3\}.$$

Compute the stabilizer of f_1.

(2) Let G and X be as defined before. Let f_2 be an element of X given by

$$f_2 := \begin{pmatrix} \{1,2\} & \{1,3\} & \{2,3\} \\ 0 & 1 & 1 \end{pmatrix}.$$

Compute the orbit of f_2; i.e., compute

$$S_3 f_2 = \{\phi(\sigma, f_2) \mid \sigma \in S_3\}.$$

Compute the stabilizer of f_2.