

EXERCISES-07

- (1) Let $n \in \mathbb{N}$ and d be a fixed positive integer. Define $p_{\leq d}(n) := \#$ of partitions of n where parts appear at most d times, and

$p_{d+1}(n) := \#$ of partitions of n with no part is divisible by k .

Show that $p_{\leq d}(n) = p_{d+1}(n)$.

- (2) Show that the number of partitions of n in which each part appears 2, 3, or 5 times equals the number of partitions of n into parts congruent to 2, 3, 6, 9, or 10 modulo 12.
- (3) Let $p(n) := 0$ for $n \in \mathbb{Z}_{<0}$. Using Euler's pentagonal number theorem, prove that for $n \in \mathbb{N}$,

$$p(n) = \sum_{r \geq 1} (-1)^{r+1} (p(n - \omega(r)) + p(n - \omega(-r))),$$

where $\omega(r) = \frac{r(3r-1)}{2}$. Compute $p(10)$ using the above recursive formula.