

1. Prove that  $\text{Lists}(n, n) = \text{Lists}(n)$  for all  $n \in \mathbb{N}$ .
2. Show that for all  $k \in \mathbb{Z}$  and  $x \in R$ ,

$$\binom{x+1}{k} = \binom{x}{k} + \binom{x}{k-1}.$$

3. Give a combinatorial proof of

$$1 + 2 + \cdots + (n-1) = \binom{n}{2}, \quad n \in \mathbb{N}.$$

4. Give a combinatorial proof of

$$\sum_{k=0}^n \binom{x}{k} \binom{y}{n-k} = \binom{x+y}{n}, \quad x, y \in R, \quad n \in \mathbb{N}.$$

5. Give a combinatorial proof of

$$(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k, \quad x \in R, \quad n \in \mathbb{N}.$$

6. For  $n, k \in \mathbb{N}$ , let  $c(n, k)$  denote the number of  $k$  element multisets on  $[n]$ .

Show that  $c(n, k) = \frac{n^{\overline{k}}}{k!}$ .