

# *Logic Programming*

## *Backtracking and Cut*

Temur Kutsia

Research Institute for Symbolic Computation  
Johannes Kepler University of Linz, Austria  
`kutsia@risc.jku.at`

# Contents

## Generating Multiple Solutions

### The "Cut"

- Confirming the Choice of a Rule

- The "Cut-fail" Combination

- Terminating a "Generate-and-Test"

- Problems with the Cut

# Finitely Many Alternatives

Simplest way: Several facts match against the question.

## Example

```
father(mary, george) .  
father(john, george) .  
father(sue, harry) .  
father(george, edward) .
```

```
?- father(X, Y) .  
X=mary, Y=george ;  
X=john, Y=george ;  
X=sue, Y=harry ;  
X=george, Y=edward
```

The answers are generated in the order in which the facts are given.

## Repeating the Same Answer

Old answers do not influence newer ones: same answer can be returned several times.

### Example

```
father(mary, george) .  
father(john, george) .  
father(sue, harry) .  
father(george, edward) .
```

```
?- father(_, X) .  
X=george ;  
X=george ;  
X=harry ;  
X=edward
```

george returned twice because George is the father of both Mary and John.

# Embedding Does Not Matter

Backtracking happens in the same way if the alternatives are embedded more deeply.

## Example

```
father(mary,george).  
father(john,george).  
father(sue,harry).  
father(george,edward).  
child(X,Y):-father(Y,X).
```

```
?- child(X,Y).  
X=george, Y=mary ;  
X=george, Y=john ;  
X=harry, Y=sue ;  
X=edward, Y=george
```

# Mixing facts and Rules

If facts and rules are mixed, the alternatives follow again in the order in which things are presented.

## Example

<code>person(adam) .</code>	<code>?- person(X) .</code>
<code>person(X) :-mother(X,Y) .</code>	<code>X=adam ;</code>
<code>person(eve) .</code>	<code>X=cain ;</code>
<code>mother(cain,eve) .</code>	<code>X=abel ;</code>
<code>mother(abel,eve) .</code>	<code>X=jabal ;</code>
<code>mother(jabal,adah) .</code>	<code>X=tubalcain ;</code>
<code>mother(tubalcain,zillah) .</code>	<code>X=eve</code>

# Multiple Goals with Multiple Solutions

More interesting case: Two goals, each with several solutions.

## Example

```
pair(X,Y):-                ?- pair(X,Y).
  boy(X),girl(Y).         X=john, Y=griselda ;
  boy(johm).              X=john, Y=ermintrude ;
  boy(marmaduke).         X=john, Y=brunhilde ;
  boy(bertram).           X=marmaduke, Y=griselda ;
  boy(charles).           X=marmaduke, Y=ermintrude ;
  girl(griselda).         X=marmaduke, Y=brunhilde ;
  girl(ermitrude).        X=bertram, Y=griselda ;
  girl(brunhilda).        ...
```

12 solutions.

# Infinite Number of Possibilities

Sometimes one might want to generate an infinite number of possibilities.

It might not be known in advance how many of them needed.

## Example

```
is_integer(0).
```

```
is_integer(X):-is_integer(Y), X is Y + 1.
```

```
?- is_integer(X).
```

```
X=0 ;
```

```
X=1 ;
```

```
X=2 ;
```

```
...
```

How does it work?



## Member and Multiple Solutions

Most rules give rise to alternative solutions if they are used for goals that contain many uninstantiated variables.

### Example

```
member(X, [X|_]).  
member(X, [_|Y]) :- member(X, Y).
```

```
?- member(a, X).  
X=[a|_G314] ;  
X=[_G313, a|_G317] ;  
X=[_G313, _G316, a|_G320] ;  
...
```

There is a way to tell PROLOG to discard choices: The "cut".

# The "Cut"

Cut (written "!") tells the system which previous choices need not to be considered again when it backtracks.

Advantages:

- ▶ The program will run faster. No time wasting on attempts to re-satisfy certain goals.
- ▶ The program will occupy less memory. Less backtracking points to be remembered.

# Example of Cut

Reference library:

- ▶ Determine which facilities are available.
- ▶ If one has an overdue book can only use the *basic facilities*.
- ▶ Otherwise can use the *general facilities*.

# Reference Library

## Example

```
facility(Pers, Fac) :-  
    book_overdue(Pers, Book), !,  
    basic_facility(Fac).  
facility(Pers, Fac) :- general_facility(Fac).  
basic_facility(reference).  
basic_facility(enquiries).  
additional_facility(borrowing).  
additional_facility(inter_library_loan).  
general_facility(X) :- basic_facility(X).  
general_facility(X) :- additional_facility(X).
```

# Reference Library

## Example

```
book_overdue('C. Watzer', book10089).  
book_overdue('A. Jones', book29907).  
...  
client('C. Watzer').  
client('A. Jones').  
...  
  
?- client(X), facility(X,Y).
```

How does it proceed?

# Reference Library

The effect of cut:

- ▶ If a client has an overdue book, then only allow her/him the basic facilities.
- ▶ Don't bother going through all the clients overdue books.
- ▶ Don't remember any other rule about facilities.

# The Effect of Cut

In general, when a cut is encountered as a goal

- ▶ The system becomes committed to all choices made since the parent goal was invoked.
- ▶ All other alternatives are discarded.
- ▶ An attempt to re-satisfy any goal between the parent goal and the cut goal will fail.

# Common Uses of Cut

Three main cases:

1. To tell the system that it found the right rule for a particular goal. *Confirming the choice of a rule.*
2. To tell the system to fail a particular goal without trying for alternative solutions. *Cut-fail combination.*
3. To tell the system to terminate the generation of alternative solutions by backtracking. *Terminate a "generate-and-test".*



# Confirming the Choice of a Rule

Typical situation:

- ▶ We wish to associate several clauses with the same predicate.
- ▶ One clause is appropriate if the arguments are of one form, another is appropriate if the arguments have another form.
- ▶ Often (but not always) these alternatives can be made disjoint by providing just the argument patterns (e.g., empty list in one clause, and a nonempty list in another.)
- ▶ If we cannot specify an exhaustive set of patterns, we may give rules for some specific argument types and give a "catchall" rule at the end for everything else.

# Confirming the Choice of a Rule

Example of the case when an exhaustive set of patterns can not be specified:

## Example

```
sum_to(1,1).
```

```
sum_to(N,Res):-  
    N1 is N - 1,  
    sum_to(N1,Res1),  
    Res is Res1 + N.
```

```
?- sum_to(5,X).  
X=15 ;
```

It loops.

# Confirming the Choice of a Rule

## What happened?

- ▶ `sum_to(1, 1)` and `sum_to(N, Res)` are not disjoint alternatives.
- ▶ `sum_to(1, 1)` matches both `sum_to(1, 1)` and `sum_to(N, Res)`.
- ▶ But if a goal matches `sum_to(1, 1)`, there is no reason why it should try the second alternative, `sum_to(N, Res)`.
- ▶ Cut the second alternative.

# Confirming the Choice of a Rule

## Example

```
sum_to(1,1):-!.
```

```
sum_to(N,Res):-  
    N1 is N - 1,  
    sum_to(N1,Res1),  
    Res is Res1 + N.
```

```
?- sum_to(5,X).
```

```
X=15 ;
```

```
No
```

## More Usual Situation

- ▶ In the previous example we could specify a pattern for the boundary case `sum_to(1, 1)`.
- ▶ Usually, it is hard to specify pattern if we want to provide extra conditions that decide on the appropriate rule.
- ▶ The previous example still loops on goals `sum_to(N, Res)` **where**  $N \leq 1$ .
- ▶ We can put this condition in the boundary case telling PROLOG to stop for such goals.
- ▶ But then the pattern can not be specified.

# Cut with Extra Conditions

## Example

```
sum_to(N,1):-N =< 1, !.
```

```
sum_to(N,Res):-  
    N1 is N - 1,  
    sum_to(N1,Res1),  
    Res is Res1 + N.
```

# Cut and Not

## General principle:

- ▶ When cut is used to confirm the choice of a rule, it can be replaced with `not`.
- ▶ `not (X)` succeeds when `X`, seen as a PROLOG goal, fails.
- ▶ Replacing cut with `not` is often considered a good programming style.
- ▶ However, it can make the program less efficient.
- ▶ Trade-off between readability and efficiency.

# Cut and Not

## Example (With Cut)

```
sum_to(1,1):-!.  
  
sum_to(N,Res):-  
    N1 is N - 1,  
    sum_to(N1,Res1),  
    Res is Res1 + N.
```

## Example (With Not)

```
sum_to(1,1).  
sum_to(N,Res):-  
    not(N=1), % N \ = 1,  
    N1 is N - 1,  
    sum_to(N1,Res1),  
    Res is Res1 + N.
```



# Cut and Not

## Example (With Cut)

```
sum_to(N,1):-N =< 1,  
    !.  
sum_to(N,Res):-  
    N1 is N - 1,  
    sum_to(N1,Res1),  
    Res is Res1 + N.
```

## Example (With Not)

```
sum_to(N,1):-N =< 1.  
sum_to(N,Res):-  
    not(N=<1), % N > 1,  
    N1 is N - 1,  
    sum_to(N1,Res1),  
    Res is Res1 + N.
```

# Double Work

not might force PROLOG to try the same goal twice:

## Example

A:  $\neg B, C$ .

A:  $\neg \text{not}(B), D$ .

B may be tried twice after backtracking.

# The "Cut-fail" Combination

`fail.`

- ▶ Built-in predicate.
- ▶ No arguments.
- ▶ Always fails as a goal and causes backtracking.

# The "Cut-fail" Combination

`fail` **after** `cut`:

- ▶ The normal backtracking behavior will be altered by the effect of `cut`.
- ▶ Quite useful combination in practice.

# The Average Taxpayer

Write a program to determine an average taxpayer.

Two cases:

- ▶ Foreigners are not average taxpayers.
- ▶ If a person is not a foreigner, apply the general criterion (whatever it is) to find out whether he or she is an average taxpayer.

# The Average Taxpayer

## Example

```
average_taxpayer(X) :-  
    foreigner(X), !, fail.  
average_taxpayer(X) :-  
    satisfies_general_criterion(X).
```

What would happen had we omitted the cut?

# The Average Taxpayer

Wrong version, without cut:

## Example (Wrong)

```
average_taxpayer(X) :-  
    foreigner(X), fail.  
average_taxpayer(X) :-  
    satisfies_general_criterion(X).
```

If there is a foreigner `widslewip` who satisfies the general criterion, the program will incorrectly answer yes on the goal

```
?- average_taxpayer(widslewip).
```

# The Average Taxpayer

We can use cut-fail combination to define `satisfies_general_criterion`.

Two cases:

- ▶ A person whose spouse earns more than a certain amount (e.g. Euro 3000) does not satisfy the criterion of being an average taxpayer.
- ▶ If this is not the case, then a person satisfies the criterion if his income is within a certain interval (e.g. more that Euro 2000 and less than Euro 3000).



# The Average Taxpayer

Clauses for `satisfies_general_criterion`.

## Example

```
satisfies_general_criterion(X) :-  
    spouse(X,Y),  
    gross_income(Y,Inc),  
    Inc > 3000,  
    !, fail.
```

```
satisfies_general_criterion(X) :-  
    gross_income(X,Inc),  
    Inc < 3000,  
    Inc > 2000.
```

# The Average Taxpayer

We can use cut-fail combination to define `gross_income`.

Two cases:

- ▶ A person who gets a pension less than certain amount (e.g. Euro 500), is considered to have no gross income.
- ▶ Otherwise, person's gross income is determined as the sum of his/her gross salary and investment income.

# The Average Taxpayer

Clauses for `gross_income`.

## Example

```
gross_income(X,Y) :-  
    receives_pension(X,P),  
    P < 500,  
    !, fail.  
gross_income(X,Y) :-  
    gross_salary(X,Z),  
    investment_income(X,W),  
    Y is Z+W.
```

## not with Cut and Fail

not can be defined in terms of cut and fail.

### Example

```
not(P) :- call(P), !, fail.  
not(P).
```

## Replacing Cut with `not`

- ▶ Cut can be replaced with `not` in cut-fail combination.
- ▶ Unlike the first use of cut, this replacement does not affect efficiency.
- ▶ However, more reorganization of the program is required.

### Example

```
average_taxpayer(X) :-  
    not(foreigner(X)),  
    not(spouse(X,Y), gross_income(Y, Inc), Inc>3000),  
    :
```

# Terminating a "Generate-and-Test"

"Generate-and-Test":

- ▶ One of the simplest AI search techniques.
- ▶ **Generate**: Generate all possible solutions to a problem.
- ▶ **Test**: Test each to see whether they are a solution.
- ▶ A possible solution is generated and then tested.
- ▶ If the test succeeds a solution is found.
- ▶ otherwise, backtrack to next possible solution.

# Tic-Tac-Toe

Tic-Tac-Toe game: Get three in a row, column, or diagonal:

X		O
O	O	
X	X	X

X	X	O
O	X	
O	X	

O		O
X	O	
X	X	O

Representation:

1	2	3
4	5	6
7	8	9

# Tic-Tac-Toe

We will show a part of the program to play Tic-Tac-Toe.

Used predicates:

- ▶ `var`: built-in predicate. `var(T)` succeeds if `T` is a free variable.
- ▶ `arg`: built-in predicate. `arg(N, T, A)` succeeds if `A` is `N`th argument of the term `T`.
- ▶ `aline`: defined predicate. Generator of possible lines. For instance, `aline([1, 5, 9])` is the following line:

X		
	X	
		X



## Part of the Program for Tic-Tac-Toe

The opponent (playing with crosses) is threatening to claim a line:

```
threatening([X,Y,Z],B,X) :-  
    empty(X,B), cross(Y,B), cross(Z,B).
```

X		
X		

	X	X

# Part of the Program

## Example

```
forced_move(Board, Sq) :-  
    aline(Squares),  
    threatening(Squares, Board, Sq),  
    !.  
  
aline([1, 2, 3]).  
aline([4, 5, 6]).  
aline([7, 8, 9]).  
aline([1, 4, 7]).  
aline([2, 5, 8]).  
aline([3, 6, 9]).  
aline([1, 5, 9]).  
aline([3, 5, 7]).
```

# Part of the Program

## Example (Cont.)

```
threatening([X,Y,Z],B,X) :-  
    empty(X,B), cross(Y,B), cross(Z,B).  
threatening([X,Y,Z],B,X) :-  
    empty(Y,B), cross(X,B), cross(Z,B).  
threatening([X,Y,Z],B,X) :-  
    empty(Z,B), cross(X,B), cross(Y,B).
```

## forced\_move

`forced_move` implements "generate-and-test":

- ▶ Moves Generated by `alignes`: All possible ways that cross can win.
- ▶ Moves Tested by `threatening`: If cross can win in the next move.
- ▶ If no forced moves are found, then the predicate fails and some other predicate would decide what move to make.

# Cut

Suppose embedded in a larger program:

- ▶ If `forced_move` successfully finds a move then `Sq` becomes instantiated to the move.
- ▶ If, later, a failure occurs (after this instantiation) `forced_move` would retry.
- ▶ Cut can prevent PROLOG to search further (which would be futile) and not waste time.
- ▶ When we look for forced moves it is only the first solution that is important.

# Problems with the Cut

Cut changes behavior of programs:

- ▶ Introducing cuts may give a correct behavior when goals are of one form.
- ▶ There is no guarantee that anything sensible will happen if goals of another form start appearing.

# Problems with the Cut

## Example

```
number_of_parents(adam,0) :- !.  
number_of_parents(eve,0) :- !.  
number_of_parents(_,2).
```

```
?- number_of_parents(eve,X).
```

```
X = 0 ;
```

```
No
```

```
?- number_of_parents(john,X).
```

```
X = 2 ;
```

```
No
```

```
?- number_of_parents(eve,2).
```

```
Yes
```

# Problems with the Cut

## Example

### Improved Version

```
number_of_parents(adam,N) :- !, N=0.
```

```
number_of_parents(eve,N) :- !, N=0.
```

```
number_of_parents(_,2).
```

```
?- number_of_parents(eve,2).
```

No

However, it will still not work properly if we give goals such as

```
?- number_of_parents(X,Y).
```