Logic Programming

Using Data Structures
Part 2

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Contents

Comparing Structures

Structure comparison:

- More complicated than the simple integers
- Have to compare all the individual components
- Break down components recursively.

Comparing Structures. `aless`

Example

`aless(X,Y)` succeeds if

- `X` and `Y` stand for atoms and
- `X` is alphabetically less than Y.

`aless(avocado, clergyman)` succeeds.
`aless(windmill, motorcar)` fails.
`aless(picture, picture)` fails.

Comparing Structures. `aless`

Success Empty word is smaller than a nonempty one.

Success The first character of the first word is alphabetically less than one of the second:
`aless(avocado, clergyman).`

Recursion The first character is the same in both. Then have to check the rest:
For `aless(lazy, leather)` check
`aless(azy, eather).`

Failure The first character of the first word is greater than the first one of the second:
`aless(book, apple).`

Failure Reach the end of both words at the same time:
`aless(apple, apple).`

Failure Run out of characters for the second word:
`aless(alphabetic, alp).`

Representation

- Transform atoms into a recursive structure.
- List of integers (ASCII codes).
- Use built-in predicate `atom_codes`:

```
?- atom_codes(alp, [97,108,112]).
yes

?- atom_codes(alp, X).
X = [97,108,112] ?
yes

?-atom_codes(X, [97,108,112]).
X = alp ?
yes
```

First Task

Convert atoms to lists:

```
atom_codes(X, XL).
atom_codes(Y, YL).
```

Compare the lists:

```
alessx(XL, YL).
```

Putting together:

```
aless(X, Y) :-
    atom_codes(X, XL),
    atom_codes(Y, YL),
    alessx(XL, YL).
```


Second Task

Compose `alessx`.

Success First word ends before second:

```
alessx([], [_|_]).
```

Success The first character in the first is alphabetically less than the the one in the second:

```
alessx([X|_], [Y|_]) :- X < Y.
```

Recursion The first character is the same in both. Then have to check the rest:

```
alessx([H|X], [H|Y]) :- alessx(X, Y).
```

What about failing cases?

Program

```
aless(X, Y):-
    atom_codes(X, XL),
    atom_codes(Y, YL),
    alessx(XL, YL).

alessx([], [_|_]).
alessx([X|_], [Y|_]):-
    X < Y.
alessx([H|X], [H|Y]):-
    alessx(X, Y).
```


Appending Two Lists

For any lists `List1`, `List2`, and `List3`
`List2` appended to `List1` is `List3` iff either

- `List1` is the empty list and `List3` is `List2`, or
- `List1` is a nonempty list and
  - the head of `List3` is the head of `List1` and
  - the tail of `List3` is `List2` appended to the tail of `List1`.

Program:

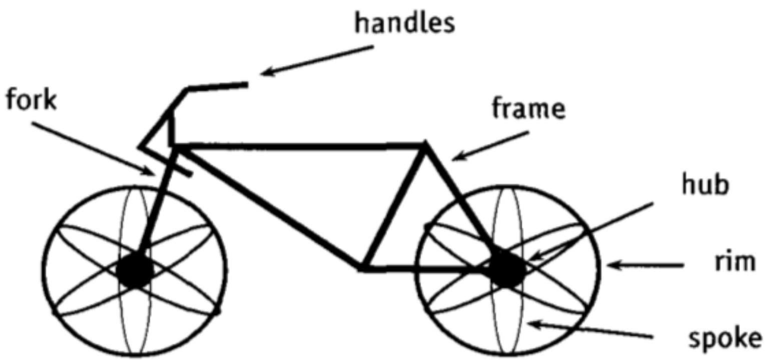
```
append([], L, L).
append([X|L1], L2, [X|L3]) :-
    append(L1, L2, L3).
```

Using append

Test	`?- append([a,b,c],[2,1],[a,b,c,2,1]).`
Total List	`?- append([a,b,c],[2,1],X).`
Isolate	`?- append(X,[2,1],[a,b,c,2,1]).`
	`?- append([a,b,c],X,[a,b,c,2,1]).`
Split	`?- append(X,Y,[a,b,c,2,1]).`

Inventory Example: Bicycle Factory

- To build a bicycle we need to know which parts to draw from the supplies.
- Each part of a bicycle may have subparts.
- Task: Construct a tree-based database that will enable users to ask questions about which parts are required to build a part of bicycle.



Parts of a Bicycle

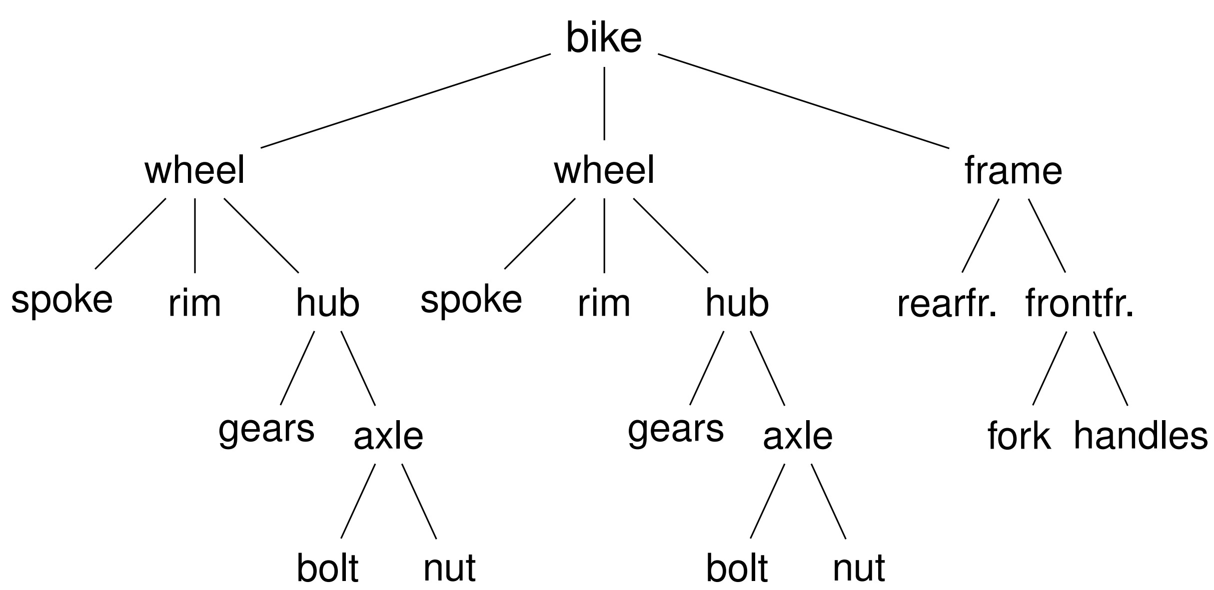
- Basic parts:

```
basicpart(rim).          basicpart(gears).
basicpart(spoke).        basicpart(bolt).
basicpart(rearframe).    basicpart(nut).
basicpart(handles).      basicpart(fork).
```

- Assemblies, consisting of a quantity of basic parts or other assemblies:

```
assembly(bike, [wheel,wheel,frame]).
assembly(wheel, [spoke,rim,hub]).
assembly(frame, [rearframe,frontframe]).
assembly(hub, [gears,axle]).
assembly(axle, [bolt,nut]).
assembly(frontframe, [fork,handles]).
```


Bike as a Tree



Program

Write a program that, given a part, will list all the basic parts required to construct it.

Idea:

1. If the part is a basic part then nothing more is required.
2. If the part is an assembly, apply the same process (of finding subparts) to each part of it.

Predicates: `partsof`

`partsof(X, Y)`: Succeeds if `X` is a part of bike, and `Y` is the list of basic parts required to construct `X`.

- Boundary condition. Basic part:
  `partsof(X, [X]) :- basicpart(X).`
- Assembly:

```
partsof(X, P) :-
    assembly(X, Subparts),
    partsoflist(Subparts, P).
```

- Need to define `partsoflist`.

Predicates: `partsoflist`

- Boundary condition. List of parts for the empty list is empty:
  `partsoflist([], []).`
- Recursive case. For a nonempty list, first find `partsof` of the head, then recursively call `partsoflist` on the tail of the list, and glue the obtained lists together:

```
partsoflist([P|Tail], Total) :-
    partsof(P, Headparts),
    partsoflist(Tail, Tailparts),
    append(Headparts, Tailparts, Total).
```

▸ The same example using accumulators

Finding Parts

```
?- partsof(bike, Parts).

Parts=[spoke,rim,gears,bolt,nut,spoke,rim,
       gears,bolt,nut,rearframe,fork,handles] ;
No


?- partsof(wheel, Parts).

Parts=[spoke, rim, gears, bolt, nut] ;
No
```

Using Intermediate Results

Frequent situation:

- Traverse a PROLOG structure.
- Calculate the result which depends on what was found in the structure.
- At intermediate stages of the traversal there is an intermediate value for the result.

Common technique:

- Use an argument of the predicate to represent the "answer so far".
- This argument is called an accumulator.

Length of a List without Accumulators

Example

`listlen(L, N)` succeeds if the length of list `L` is `N`.

- Boundary condition. The empty list has length 0:
  `listlen([], 0).`
- Recursive case. The length of a nonempty list is obtained by adding one to the length of the tail of the list.

```
listlen([H|T], N) :-
     listlen(T, N1),
     N is N1 + 1.
```

Length of a List with an Accumulator

Example

`lenacc(L, A, N)` succeeds if the length of list `L`, when added the number `A`, is `N`.

- Boundary condition. For the empty list, the length is whatever has been accumulated so far, i.e. `A`:
  `lenacc([], A, A).`
- Recursive case. For a nonempty list, add 1 to the accumulated amount given by `A`, and recur to the tail of the list with a new accumulator value `A1`:

```
lenacc([H|T], A, N) :-
    A1 is A + 1,
    lenacc(T, A1, N).
```


Length of a List with an Accumulator, Cont.

Example

Complete program:

```
listlenacc(L, N) :-
    lenacc(L, 0, N).

lenacc([], A, A).
lenacc([H|T], A, N) :-
    A1 is A + 1,
    lenacc(T, A1, N).
```

Computing List Length

Example (Version without Accumulator)

```
listlen([a,b,c], N).
listlen([b,c], N1), N is N1 + 1.
listlen([c], N2), N1 is N2 + 1, N is N1 + 1.
listlen([], N3), N2 is N3 + 1, N1 is N2 + 1,
N is N1 + 1.
N2 is 0 + 1, N1 is N2 + 1, N is N1 + 1.
N1 is 1 + 1, N is N1 + 1.
N is 2 + 1.

N = 3
```


Computing List Length

Example (Version with an Accumulator)

```
listlenacc([a,b,c], N).
lenacc([a,b,c], 0, N).
A1 is 0+1, lenacc([b,c], A1, N).
lenacc([b,c], 1, N).
A2 is 1+1, lenacc([c], A2, N).
lenacc([c], 2, N).
A3 is 2+1, lenacc([], A3, N).
lenacc([], 3, N).

N = 3
```

List as an Accumulator

- Accumulators need not be integers.
- If a list is to be produced as a result, an accumulator will hold a list produced so far.
- Wasteful joining of structures avoided.

Example (Reversing Lists)

```
reverse(List, Rev) :-
    rev_acc(List, [], Rev).

rev_acc([], Acc, Acc).
rev_acc([X|T], Acc, Rev) :-
    rev_acc(T, [X|Acc], Rev).
```


Bicycle Factory

Recall how parts of bike were found. Inventory example
`partsoflist` has to find the parts coming from the list `[wheel,wheel,frame]`:

- Find parts of `frame`.
- Append them to `[]` to find parts of `[frame]`.
- Find parts of `wheel`.
- Append them to the parts of `[frame]` to find parts of `[wheel,frame]`.
- Find parts of `wheel`.
- Append them to the parts of `[wheel,frame]` to find parts of `[wheel,wheel,frame]`.

Wasteful!

Bicycle Factory

Improvement idea: Get rid of append. Use accumulators.
`partsacc(X, A, P)`: parts of `X`, when added to `A`, give `P`.

```
partsof(X, P) :- partsacc(X, [], P).

partsacc(X, A, [X|A]) :- basicpart(X).
partsacc(X, A, P) :-
    assembly(X, Subparts),
    partsacclist(Subparts, A, P).

partsacclist([], A, A).
partsacclist([P|Tail], A, Total) :-
    partsacc(P, A, Headparts),
    partsacclist(Tail, Headparts, Total).
```


Difference Structures

Compute parts of wheel without and with accumulator:

Example (Without Accumulator)

```
?- partsof(wheel, P).
X = [spoke, rim, gears, bolt, nut] ;
No
```

Example (With Accumulator)

```
?- partsof(wheel, P).
X = [nut, bolt, gears, rim, spoke] ;
No
```

Reversed order.

Difference Structures

How to avoid wasteful work and retain the original order at the same time?

Difference structures.

Open Lists and Difference Lists

- Consider the list `[a,b,c|Ho]`.
- The structure of the list is known up to a point.
- If, at some point, `Ho` is unbound then we have an open list.
- Informally, `Ho` is a called a “hole”.

Open Lists and Difference Lists

- Unify `Ho` with `[d,e]`:

```
?- List=[a,b,c|Ho], Ho=[d,e].
List=[a,b,c,d,e]
```

- We started with an open list and “filled” in the hole with the structure.

Open Lists and Difference Lists

- The result of filling in the hole in an open list with a “proper” list is a “proper” list.
- What happens if we instantiate the hole with an open list?
- The result will be an open list again:

```
?- List=[a,b,c|Ho], Ho=[d,e|Y].
?- List=[a,b,c,d,e|Y].
```

Open Lists and Difference Lists

- Filling in the hole with a proper list, again:
- `?- List=[a,b,c|Ho], Ho=[d,e].`
- `?- List=[a,b,c,d,e].`
- Is not it the same as `append([a,b,c],[d,e],List)`?

open_append

- We can define `append` in terms of “hole filling”.
- Assume the first list is given as an open list.
- Define a predicate that fills in the hole with the second list.
- A naive and limited way of doing this:

```
open_append([H1,H2,H3|Hole],L2):-Hole=L2.
?- List=[a,b,c|Ho], open_append(List,[d,e]).
    List=[a,b,c,d,e]
    Ho=[d,e]
```

- Improvement is needed: This version assumes having a list with three elements and the hole.

Improvement Idea

- One often wants to say about open lists something like

  "take the open list and fill in the hole with ..."

- Hence, one should know both an open list and a hole.
- Idea for list representation: Represent a list as an open list together with the hole.
- Such a representation is called a difference list.
- Example: The difference list representation of the list `[a,b,c]` is the pair of terms `[a,b,c|X]` and `X`.

diff_append

- Difference append:

```
diff_append(OpenList, Hole, L2) :- Hole=L2.
```

```
?- List=[a,b,c|Ho], diff_append(List,Ho,[d,e]).
    List=[a,b,c,d,e]
    Ho=[d,e]
```

- Compare to the `open_append`:

```
open_append([H1,H2,H3|Hole], L2) :- Hole=L2.
```

```
?- List=[a,b,c|Ho], open_append(List,[d,e]).
    List=[a,b,c,d,e]
    Ho=[d,e]
```

Difference Lists

- Introduce a notation for difference lists.
- Idea: We are usually interested the open list part of difference list, without the hole.
- From the pair `[a,b,c|Ho]` and `Ho` we are interested in `[a,b,c]`.
- “Subtracting” the hole `Ho` from the open list `[a,b,c|Ho]`.
- `[a,b,c|Ho]-Ho`.
- The `-` has no interpreted meaning. Instead one could define any operator to use there.

`diff_append`. Version 2

- `diff_append(OpenList-Hole, L2) :- Hole=L2.`

  ```
  ?- DList=[a,b,c|Ho]-Ho,
     diff_append(DList,[d,e]).

      DList=[a,b,c,d,e]-[d,e]
      Ho=[d,e]
  ```

- Has to be improved again: We are not interested in the “filled hole” in the instantiation of `Ho` hanging around.

diff_append. Version 3

- Let diff_append return the open list part of the first argument:

```
diff_append(OpenList-Hole, L2, OpenList) :-
    Hole=L2.
```

```
?- DList=[a,b,c|Ho]-Ho,
   diff_append(Dlist,[d,e],Ans).
```

```
    Dlist=[a,b,c,d,e]-[d,e]
    Ho=[d,e]
    Ans=[a,b,c,d,e]
```

- It is better now. Ans looks as we would like to.
- Still, there is a room for improvement: The diff_append
  - takes a difference list as its first argument,
  - a proper list as its second argument, and
  - returns a proper list.
- Let's make it more uniform.

diff_append. Version 3

- Better, but not the final approximation: diff_append takes two difference lists and returns an open list:

```
diff_append(
    OpenList1-Hole1, OpenList2-Hole2, OpenList1
) :-
    Hole1=OpenList2.
```

```
?- Dlist=[a,b,c|Ho]-Ho,
   diff_append(Dlist,[d,e|Ho1]-Ho1,Ans).
```

```
   Dlist=[a,b,c,d,e|Ho1]-[d,e|Ho1]
   Ho=[d,e|Ho1]
   Ans=[a,b,c,d,e|Ho1]
```

- We have returned an open list but we want a difference list.
- The first list has gained the hole of the second list.
- All we need to ensure is that we return the hole of the second list.

`diff_append`. Version 3

- Return the hole of the second list as well:

```
diff_append(
    OpenList1-Hole1,
    OpenList2-Hole2,
    OpenList1-Hole2
) :-
    Hole1=OpenList2.
```

```
?- DList=[a,b,c|Ho]-Ho,
   diff_append(DList,[d,e|Ho1]-Ho1,Ans).

   DList=[a,b,c,d,e|Ho1]-[d,e|Ho1]
   Ho=[d,e|Ho1]
   Ans=[a,b,c,d,e|Ho1]-Ho1
```

- We have returned an difference list.
- Now we can recover the proper list we want:

```
?- DList=[a,b,c|Ho]-Ho,
   diff_append(DList,[d,e|Ho1]-Ho1,Ans-[]).

   Ans=[a,b,c,d,e]
```


`diff_append`. Version 4

`diff_append` can be made more compact:

```
diff_append(
    OpenList1-Hole1,
    Hole1-Hole2,
    OpenList1-Hole2
).
```

`diff_append`. Usage

- Add an element at the end of a list:

```
add_to_back(L-H, El, Ans) :-
    diff_append(L-H, [El|H1]-H1, Ans-[]).
?- add_to_back([a,b,c|H]-H, e, Ans).
   H = [e]
   Ans = [a,b,c,e]
```


Difference Structures

Both accumulators and difference structures use two arguments to build the output structure.

Accumulators: the “result so far” and the “final result”.

Difference structures: the (current approximation of the) “final result” and the “hole in there where the further information can be put”.

Bicycle Factory

Use holes.

```
partsof(X, P) :-
    partshole(X, P-Hole),
    Hole=[].

partshole(X, [X|Hole]-Hole) :-
    basicpart(X).
partshole(X, P-Hole) :-
    assembly(X, Subparts),
    partsholelist(Subparts, P-Hole).

partsholelist([], Hole-Hole).
partsholelist([P|Tail], Total-Hole) :-
    partshole(P, Total-Hole1),
    partsholelist(Tail, Hole1-Hole).
```


Bicycle Factory. Detailed View

```
partsof(X, P) :-
    partshole(X, P-Hole),
    Hole=[].
```

- `partshole(X, P-Hole)` builds the result in the second argument `P` and returns in `Hole` a variable.
- Since `partsof` calls `partshole` only once, it is necessary to terminate the difference list by instantiating `Hole` with `[]`. (Filling the hole.)
- Alternative definition of `partsof`:
  `partsof(X, P) :- partshole(X, P-[]).`
  It ensures that the very last hole is filled with `[]` even before the list is constructed.

Bicycle Factory. Detailed View

```
partshole(X, [X|Hole]-Hole) :-
    basicpart(X).
```

- It returns a difference list containing the object (basic part) in the first argument.
- The hole remains open for further instantiations.

Bicycle Factory. Detailed View

```
partshole(X, P-Hole):-
    assembly(X, Subparts),
    partsholelist(Subparts, P-Hole).
```

- Finds the list of subparts.
- Delegates the traversal of the list to `partsholelist`.
- The difference list `P-Hole`is passed to `partsholelist`.

Bicycle Factory. Detailed View

```
partsholelist([P|Tail], Total-Hole) :-
    partshole(P, Total-Hole1),
    partsholelist(Tail, Hole1-Hole).
```

- `partshole` starts building the `Total` list, partially filling it with the parts of `P`, and leaving a hole `Hole1` in it.
- `partsholelist` is called recursively on the `Tail`. It constructs the list `Hole1` partially, leaving a hole `Hole` in it.
- Since `Hole1` is shared between `partshole` and `partsholelist`, after getting instantiated in `partsholelist` it gets also instantiated in `partshole`.
- Therefore, at the end `Total` consists of the portion that `partshole` constructed, the portion of `Hole1` `partsholelist` constructed, and the hole `Hole`.