Logic Programming

The Basics

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Contents

PROLOG

Used to solve problems involving

- objects, and
- relationships between objects.

Relationships

Example

John owns the book

- The relationship: *ownership*
- The objects: *book, John*

Directional:

- John owns the book
- Not: The book owns John

Questions

Example

Does John own the book?

Asks a question about a relationship already established.

Rules

Describe Relationships Using other Relationships.

Example

Two people are sisters if they are both female and have the same parents.

Gives a definition of one relationship given other relationships.

- Both must be females.
- Both must have the same parents.
- If two people satisfy these rules, then they are sisters (according to our simplified relationship)

Programming in PROLOG

- Declaring Facts about objects and their relationships.
- Defining Rules about objects and their relationships.
- Asking Questions about objects and their relationships.

PROLOG

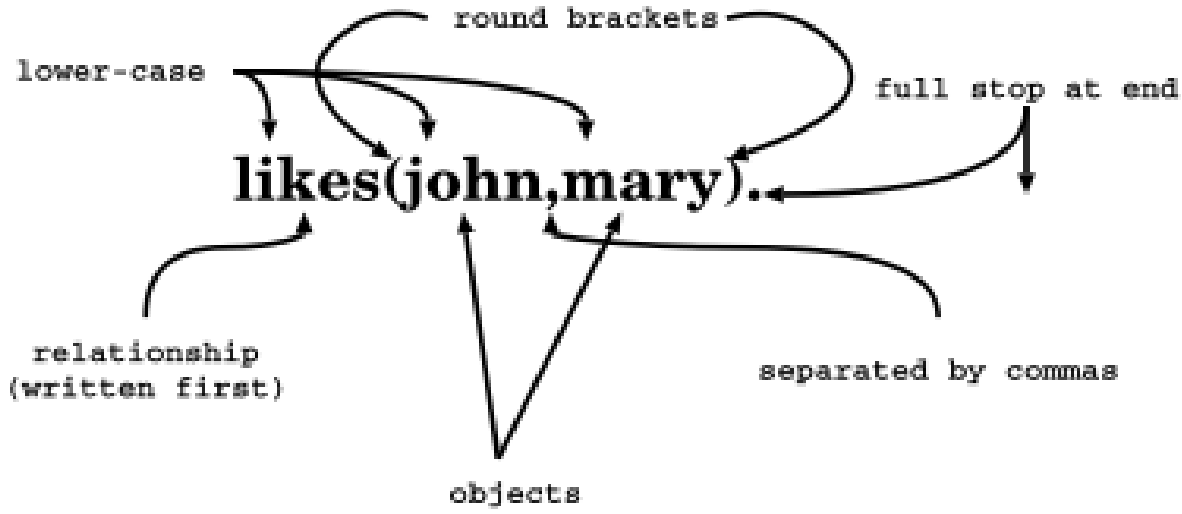
- Program can be thought of as a storehouse of facts and rules.
- Conversational Language: The user can ask questions about the set of facts and rules in the PROLOG program.

PROLOG

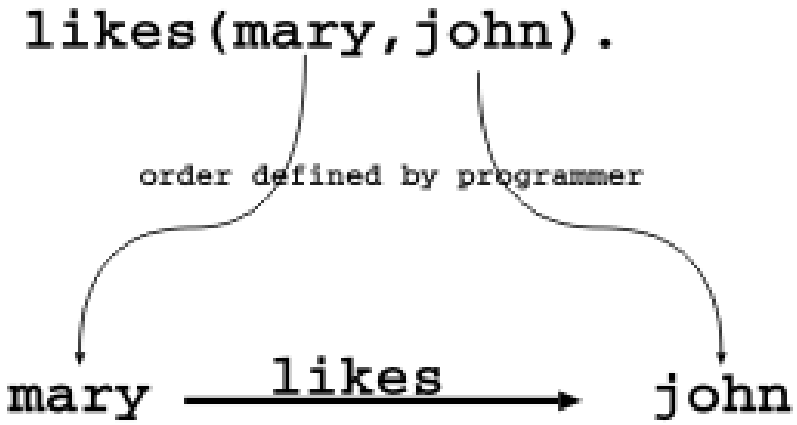
Sisters Example:

- A rule defining sisters and the facts about the people involved.
- The user would ask:
  Are these two people sisters?
- The system would answer
  yes (true) or **no** (false)

Parts of Fact



Order of Objects



The fact says nothing
about how john likes mary

Examples of Facts

Example

Gold is valuable.

```
valuable(gold)
```

Jane is a female.

```
female(jane)
```

John owns some gold.

```
owns(john, gold)
```

John is the father of Mary.

```
father(john, mary)
```

Examples of Facts

Example

Gold is valuable.

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valuable(gold)
```

Jane is a female.

```
female(jane)
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John owns some gold.

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owns(john, gold)
```

John is the father of Mary.

```
father(john, mary)
```

Are these expressions really facts? Is there anything missing?

Interpretation of Names

The name refers to an object.

- Semantic Meaning: Given by the programmer.
- Syntactic Meaning: a set of characters, as PROLOG sees it.

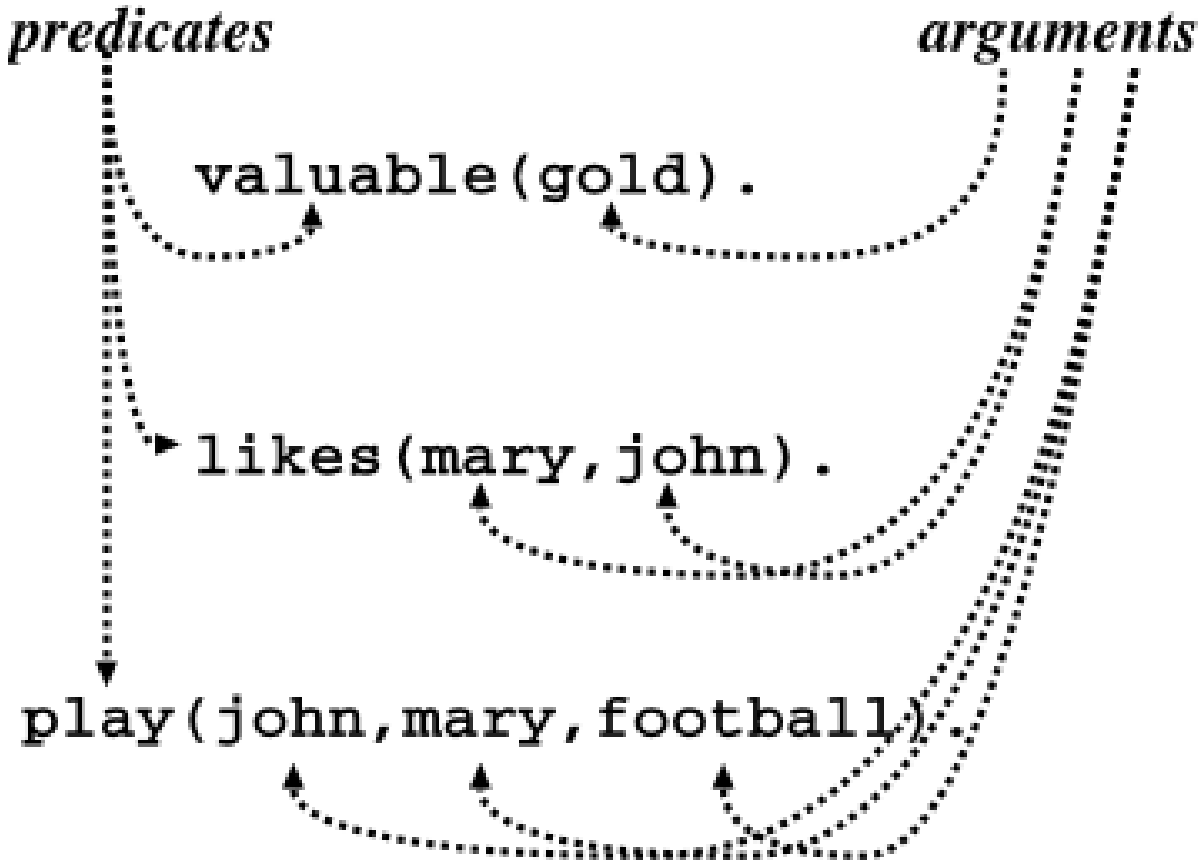
Interpretation of Names

Name refers to an object.

- Name `gold` can refer to:
  - a particular lump of gold, or
  - the chemical element Gold having atomic number 79.
- `valuable(gold)` can mean:
  - that particular lump of gold, named `gold`, is valuable, or
  - the chemical element Gold, named `gold`, is valuable.

The programmer decides (in her usage) the meaning.

Fact Terminology



Database

Definition

In PROLOG, database is a collection of facts.

- PROLOG draws its knowledge from these facts.
- The programmer is responsible for their accuracy.

Questions

- The database contains the facts from which the questions are answered.
- A question can look exactly like a fact:
  `owns(mary, book).`
- The difference is in which mode one is in.

Questions

In the interactive question mode (indicated by the question mark and dash `?-`):

- Question: `?- owns(mary, book).`
- Meaning:
  - If `mary` is interpreted as a person called Mary, and `book` is interpreted as some particular book, then
  - `?- owns(mary, book).` means: Does Mary own the book?

Database Search

Example

Facts in the database:

```
likes(joe, fish).
likes(joe, mary).
likes(mary, book).
likes(john, book).
```

Questions:

```
?- likes(joe, money).
no
?- likes(joe, mary).
yes
?- king(john, france).
no
```

Knowledge

The questions are always answered with respect to the database.

Example

Facts in the database:

```
human(socrates).
human(aristotle).
athenian(socrates).
```

Question:
Is Socrates Greek?

```
?- greek(socrates).
```

The answer with respect to this database is **No**.

Questions

Up until now questions just reflect exactly the database.

Does Mary like the book?

```
?- likes(mary, book).
```

More Interesting Question:
What objects does Mary like?

Variables.

Variables

Tiresome to ask about every object:

```
likes(john, this).
likes(john, that).
```

Better to ask:
What does John like?
or
Does John like X?
(i.e. use variables)

Question With Variables

Does John like X?

```
?- likes(john, X).
```

or

```
?- likes(john, SomethingThatJohnLikes).
```

`X` and `SomethingThatJohnLikes` are variables.

Variable begins with a capital letter.

PROLOG Answer

Database:

```
likes(john, flowers).
```

Question:

```
?- likes(john, X).
```

PROLOG answers:

```
X=flowers
```

Many Answers

Database:

```
likes(john, flowers).
likes(john, mary).
likes(paul, mary).
```

Question:

```
?- likes(john, X).
```

PROLOG answers:

```
X=flowers
```

and the user acknowledges

```
X=mary
```

and the user acknowledges

```
no
```

Placemarker

- The first match is found: `X=flowers`.
- The user acknowledges.
- From that place on the next match is found (the search continues).
- From the place of the last instantiation no more match was found.
- Thus answer: `no`.

Conjunctions

More Complicated Relationships:

Does Mary like John and does John like Mary?

Both Conditions must be fulfilled.

Conjunctions

Comma means Conjunction:

```
?- likes(john, mary), likes(mary, john).
```

```
likes(mary, food).
likes(mary, wine).
likes(john, wine).
likes(john, mary).
```

Answer: `no`

A match for `likes(john, mary)`
but none for `likes(mary, john)`

Conjunctions with Variables

Is there anything that both mary and john like?

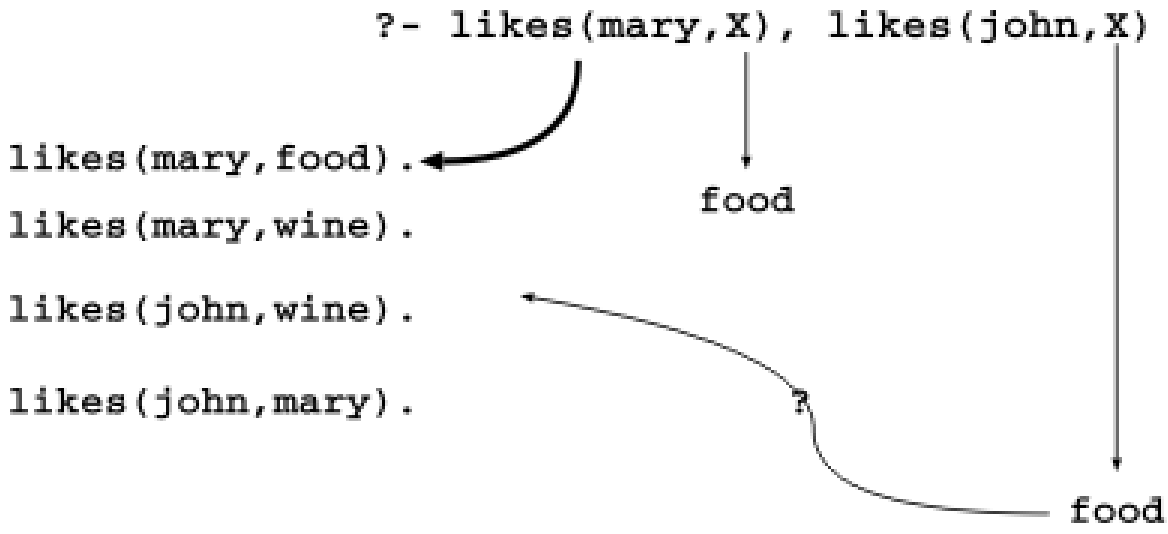
Find out what Mary likes and then see if John likes it.

```
?- likes(mary, X), likes(john, X).
```

Backtracking

- Find match for the first goal.
- Then see if it matches the second.
- If not, find another match for the first.
- See if this matches the second.
- etc.

Match First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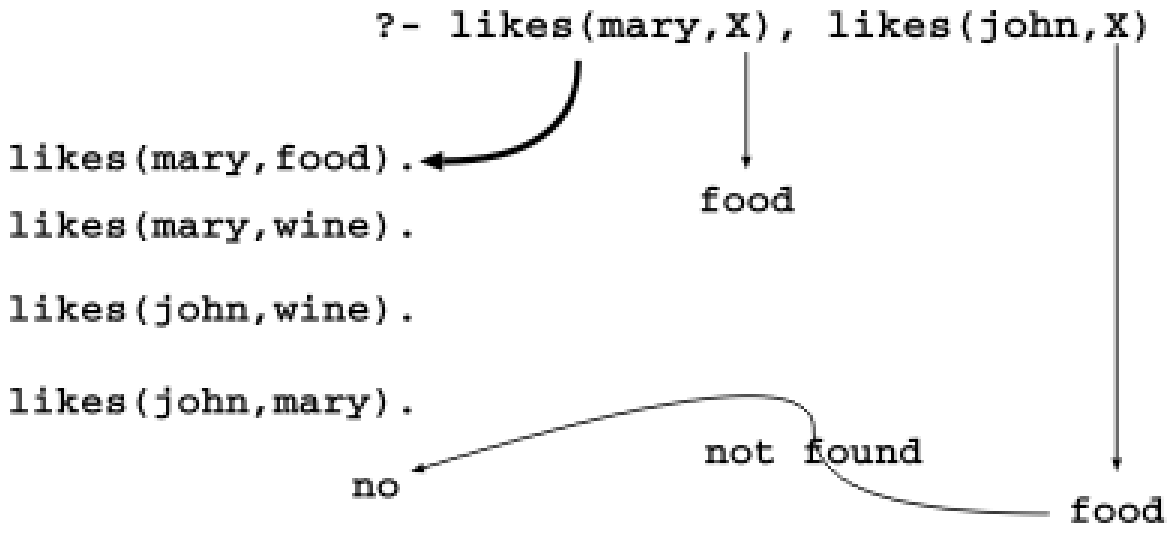
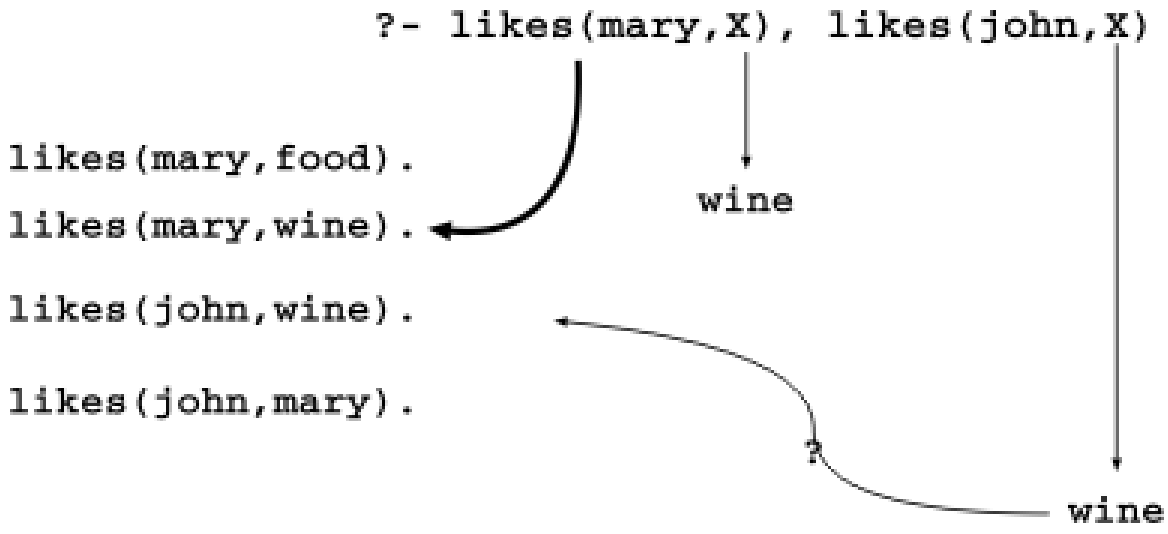
likes(mary,food).
likes(mary,wine).
likes(john,wine).
likes(john,mary).
food
?
food

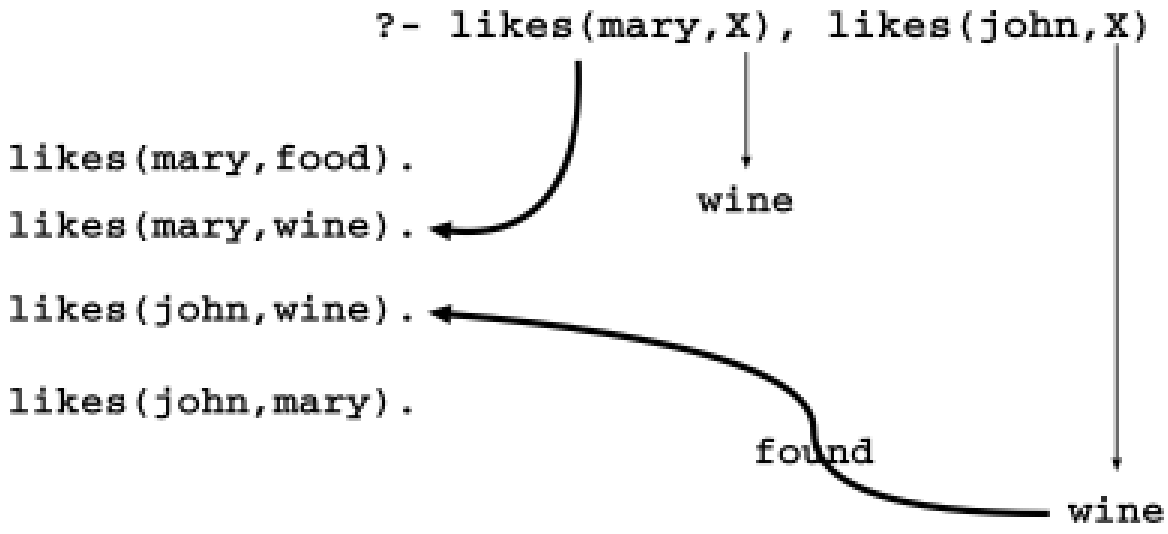
Match Second



Backtrack


?- likes(mary,X), likes(john,X)
likes(mary,food).
likes(mary,wine).
likes(john,wine).
likes(john,mary).
wine
?
wine

Success


?- likes(mary,X), likes(john,X)
likes(mary,food).
likes(mary,wine).
likes(john,wine).
likes(john,mary).
wine
found
wine

Rules

- How to express that John likes all people?

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- How to express that John likes all people?
- Listing all people?

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  - `likes(john, alfred).`

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  - `likes(john, alfred).`
  - `likes(john, bertrand).`
  - `likes(john, charles).`
  - `likes(john, david).`
  - etc.

Rules

- How to express that John likes all people?
- Listing all people?
  - `likes(john, alfred).`
  - `likes(john, bertrand).`
  - `likes(john, charles).`
  - `likes(john, david).`
  - etc.
- Not feasible. More compact way: Using rules.
  John likes any object provided it is a person.

Rule Examples

- Rules state Dependence:
  I use an umbrella **if** there is rain.

Rule Examples

- Rules state Dependence:
  I use an umbrella **if** there is rain.
- Rules Define:
  X is a bird **if** X is an animal and X has feathers.

Formulating Rules

- John likes anyone who likes wine.

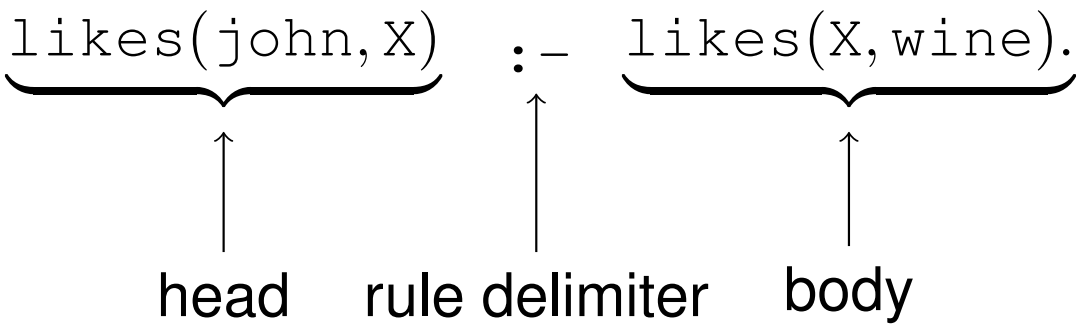
Formulating Rules

- John likes anyone who likes wine.
- John likes any something if it likes wine.

Formulating Rules

- John likes anyone who likes wine.
- John likes any something if it likes wine.
- John likes X if X likes wine.

Rule Syntax



Variable Scope

The occurrences of `X` within a rule:

```
likes(john, X) :-likes(X, wine),
                 likes(X, food).
```

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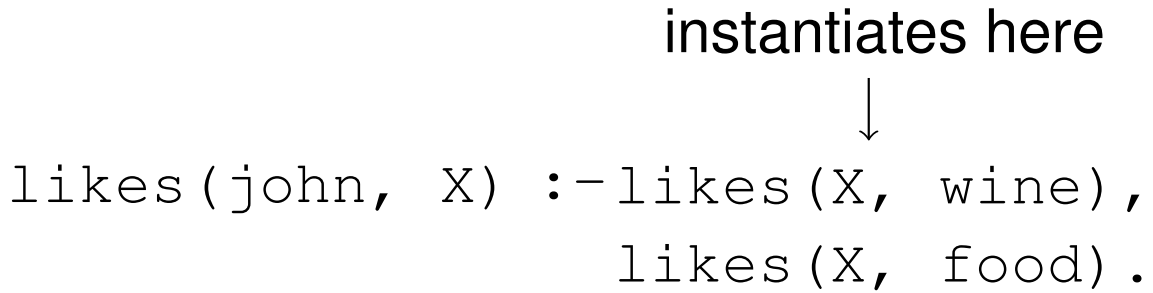

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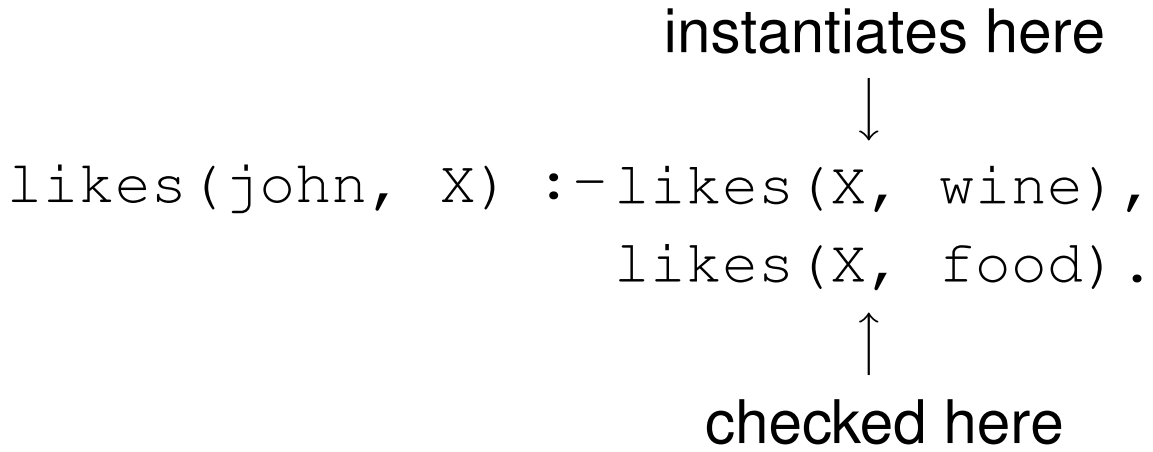

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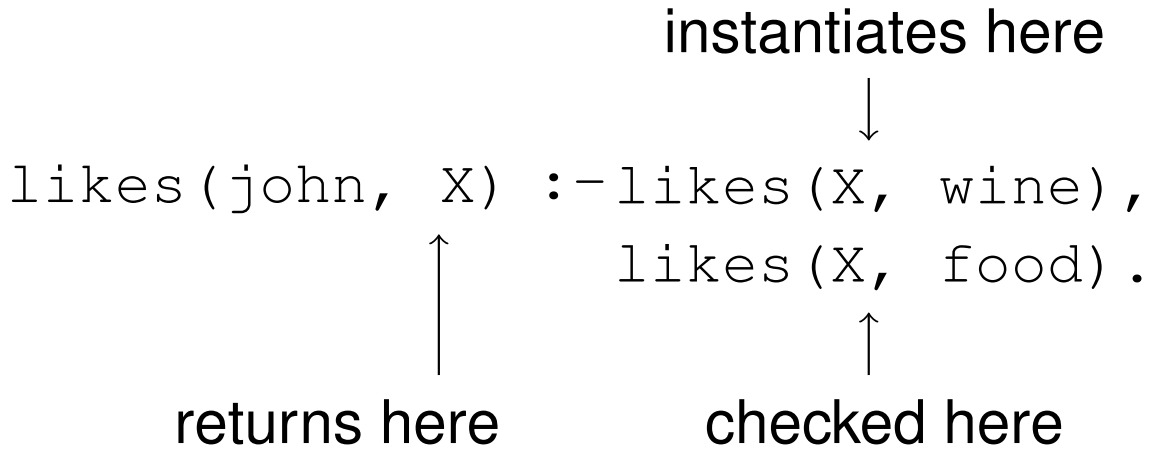

Royal Parents

Example

- The parents of X are Y and Z.
- Y is the mother.
- Z is the father.

Database:

```
male(albert).
male(edward).
female(alice).
female(victoria).
parents(edward, victoria, albert).
parents(alice, victoria, albert).
```

Sisters

Example

X is a sister of Y if:

Sisters

Example

X is a sister of Y if:

- X is female,

Sisters

Example

X is a sister of Y if:

- X is female,
- X has parents M and F,

Sisters

Example

X is a sister of Y if:

- X is female,
- X has parents M and F,
- Y has parents M and F.

Sisters

Example

X is a sister of Y if:

- X is female,
- X has parents M and F,
- Y has parents M and F.

Rule:

```
sister(X, Y) :-
    female(X),
    parents(X, M, F),
    parents(Y, M, F).
```

Sisters Question

Rule:

```
sister(X, Y) :-
    female(X),
    parents(X, M, F),
    parents(Y, M, F).
```

Question:

```
sister(alice, edward).
```

Sisters Question

Rule:

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sister(X, Y) :-
    female(X),
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Question:

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sister(alice, edward).
```

- The question (goal) matches the head of the rule, if one replaces `X` with `alice` and `Y` with `edward`.

Sisters Question

Rule:

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sister(X, Y) :-
    female(X),
    parents(X, M, F),
    parents(Y, M, F).
```

Question:

```
sister(alice, edward).
```

- The question (goal) matches the head of the rule, if one replaces `X` with `alice` and `Y` with `edward`.
- The instance of the body becomes a new goal:

```
female(alice),
parents(alice, M, F),
parents(edward, M, F).
```

Is Alice Edward's Sister?

```
sister(alice,edward)
```

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
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sister(alice,edward)

X0=alice,
Y0=edward

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7 sister(alice,edward)
                 |  X0=alice,
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                 ↓
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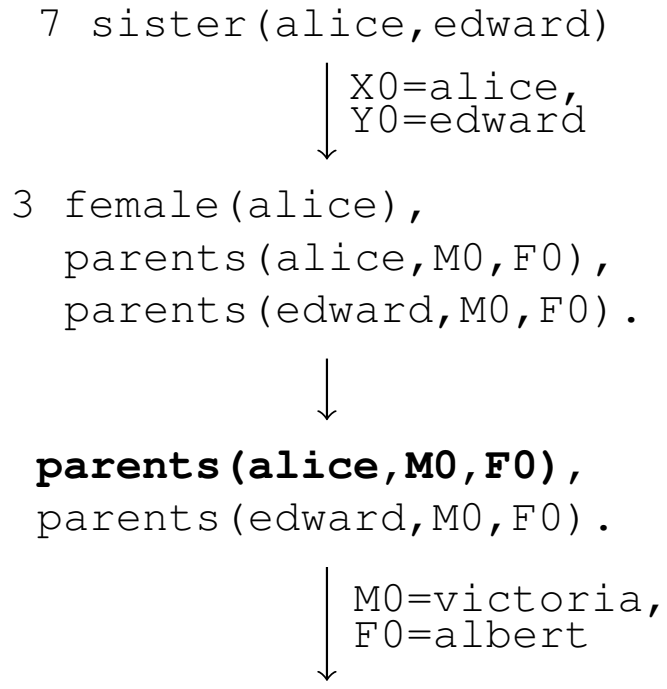

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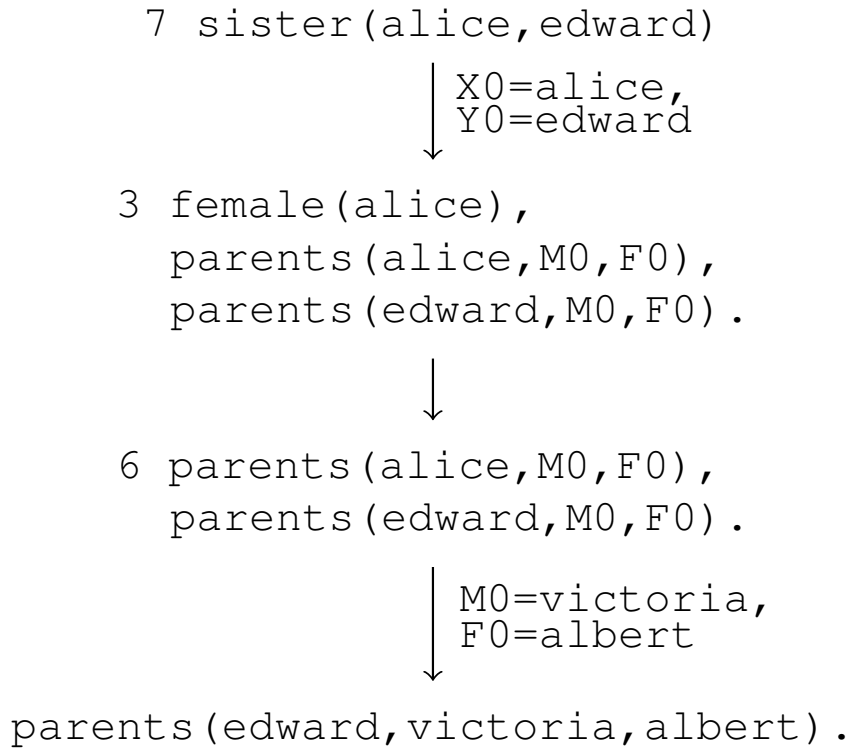

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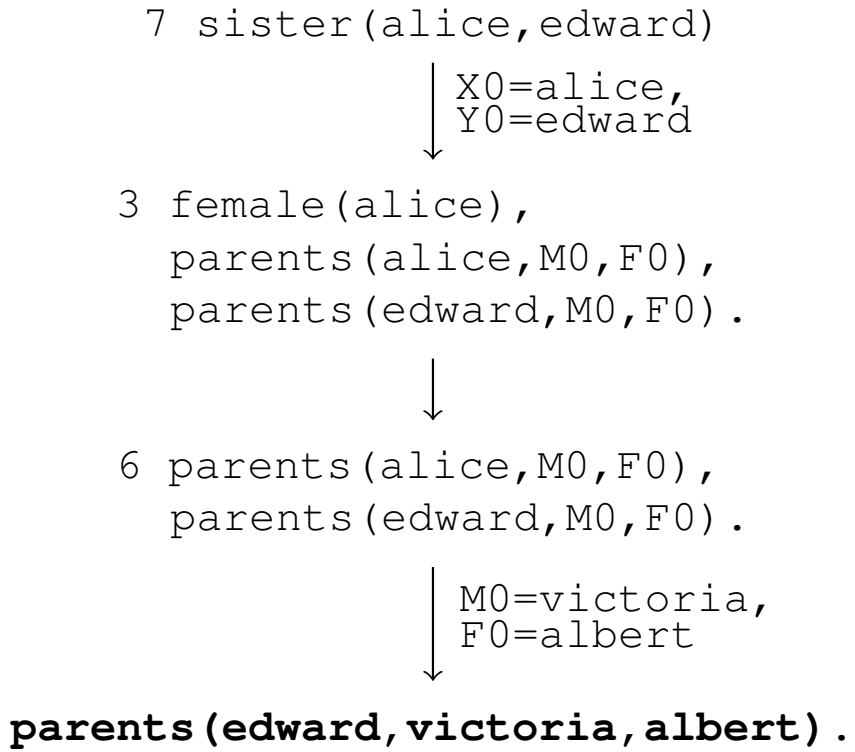

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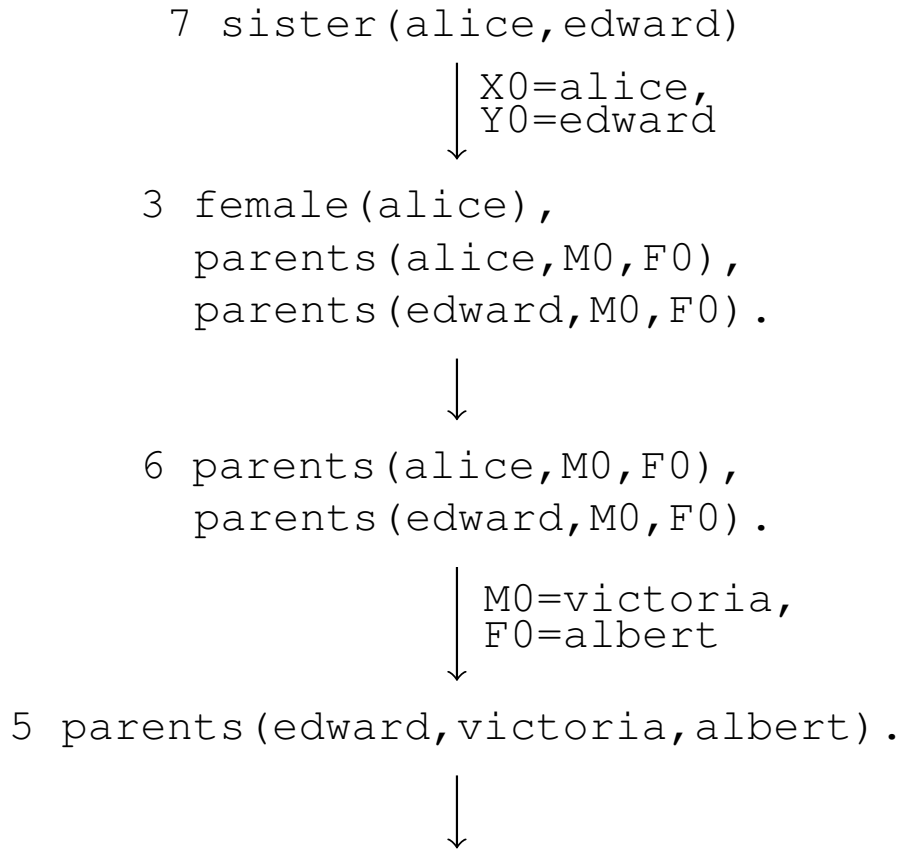
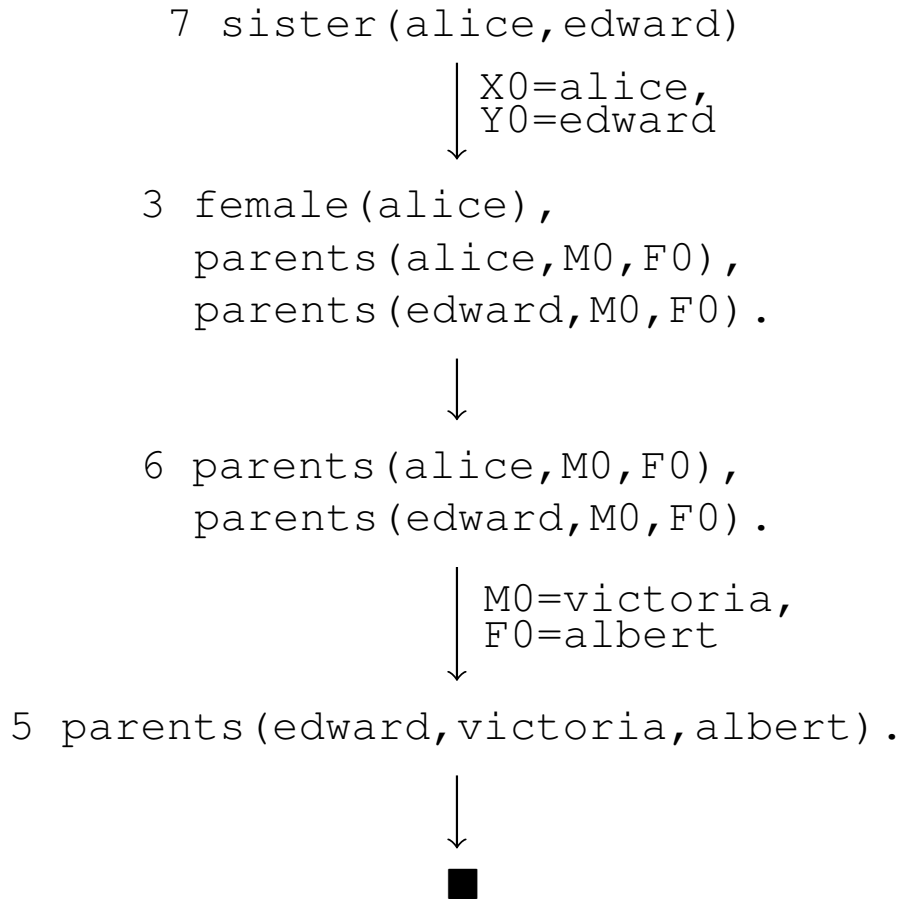
      parents(Y, M, F).
```



Is Alice Edward's Sister?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```



Who's Sister Is Alice?

```
sister(alice,X)
```

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```

Who's Sister Is Alice?

sister(alice,X)

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```

Who's Sister Is Alice?

sister(alice,X)

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```

Who's Sister Is Alice?

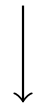
sister(alice,X)

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X0, Y0):-
      female(X0),
      parents(X0, M0, F0),
      parents(Y0, M0, F0).
```

Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X0, Y0):-
      female(X0),
      parents(X0, M0, F0),
      parents(Y0, M0, F0).
```

sister(alice,X)



X0=alice,
Y0=X

Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X0, Y0):-
      female(X0),
      parents(X0, M0, F0),
      parents(Y0, M0, F0).
```

```
7 sister(alice,X)
        |  X0=alice,
        |  Y0=X
        ↓
female(alice),
parents(alice,M0,F0),
parents(X,M0,F0).
```

Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```

```
7 sister(alice,X)
        |  X0=alice,
        |  Y0=X
        ↓
female(alice),
parents(alice,M0,F0),
parents(X,M0,F0).
```

Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```

```
7 sister(alice,X)
        |  X0=alice,
        |  Y0=X
        ↓
female(alice),
parents(alice,M0,F0),
parents(X,M0,F0).
```

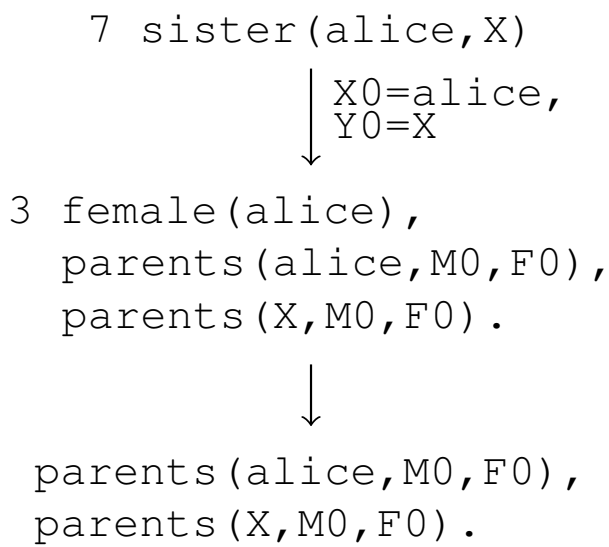
Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```

```
7 sister(alice,X)
        |
        ↓  X0=alice,
           Y0=X
female(alice),
parents(alice,M0,F0),
parents(X,M0,F0).
        |
        ↓
```

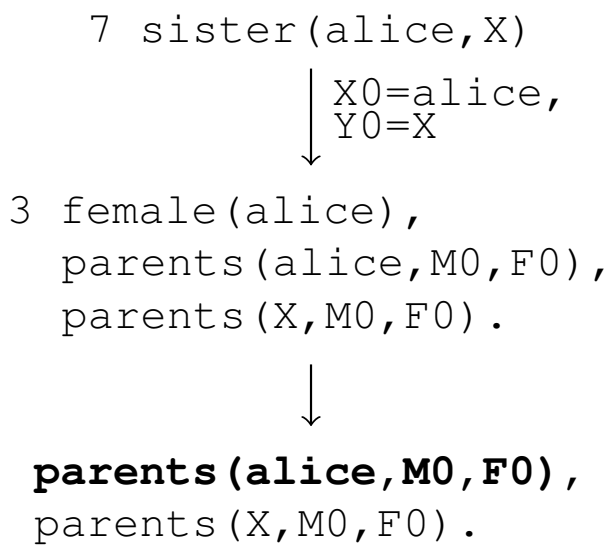
Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```



Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```



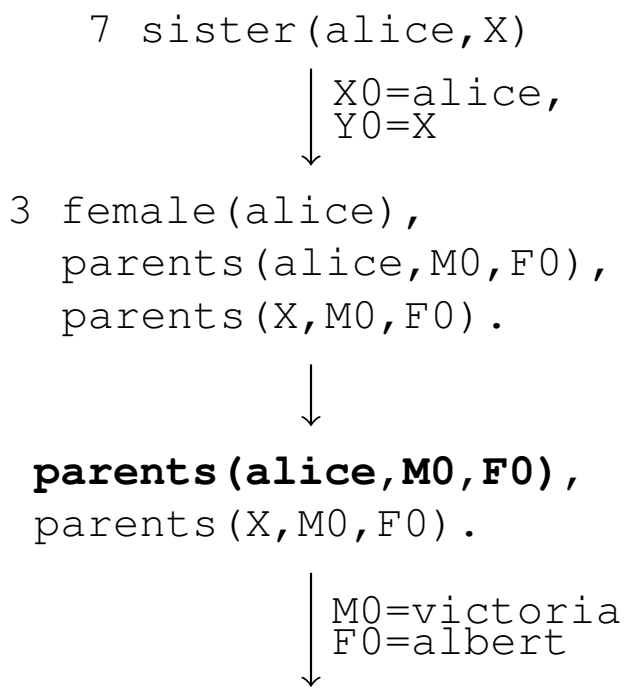
Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```

```
7 sister(alice,X)
          |
          | X0=alice,
          | Y0=X
          ↓
3 female(alice),
  parents(alice,M0,F0),
  parents(X,M0,F0).
          ↓
parents(alice,M0,F0),
parents(X,M0,F0).
```

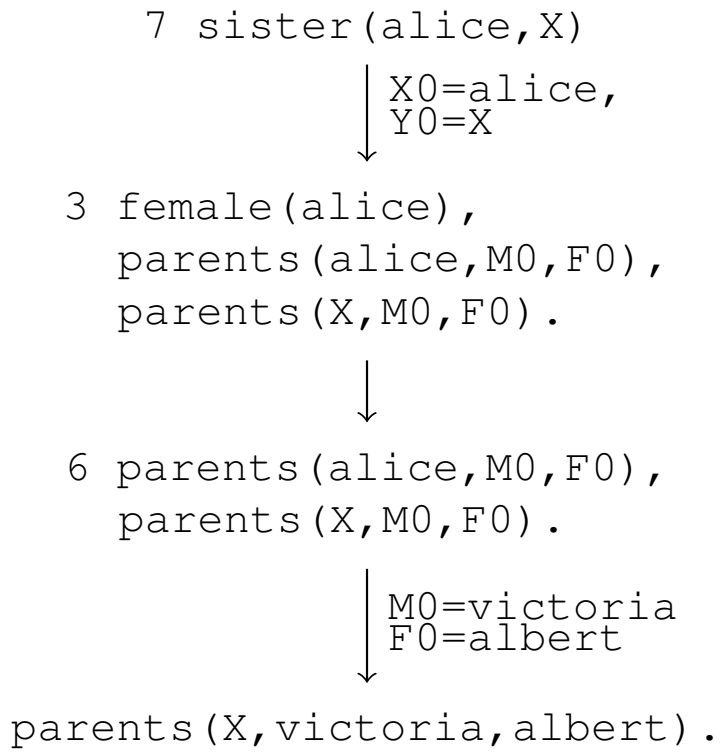
Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```



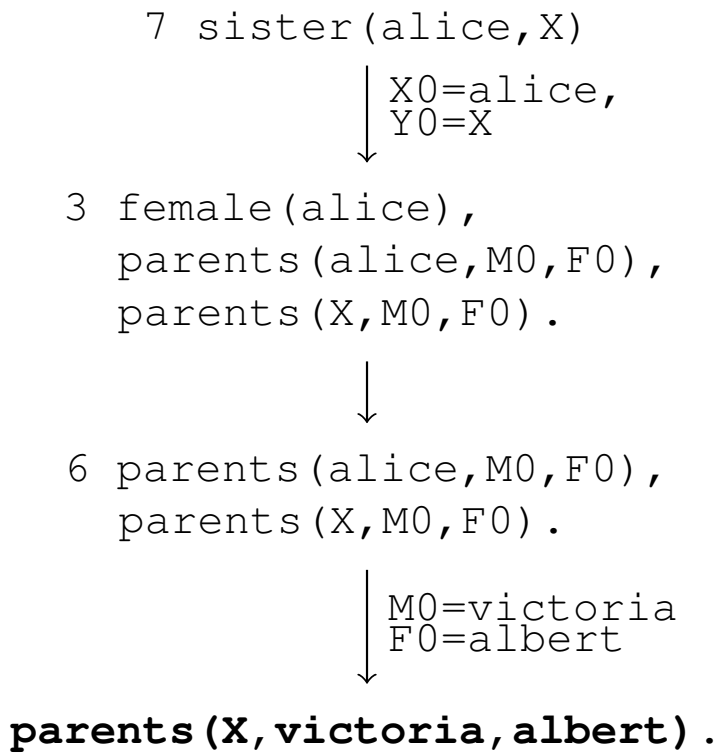
Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```



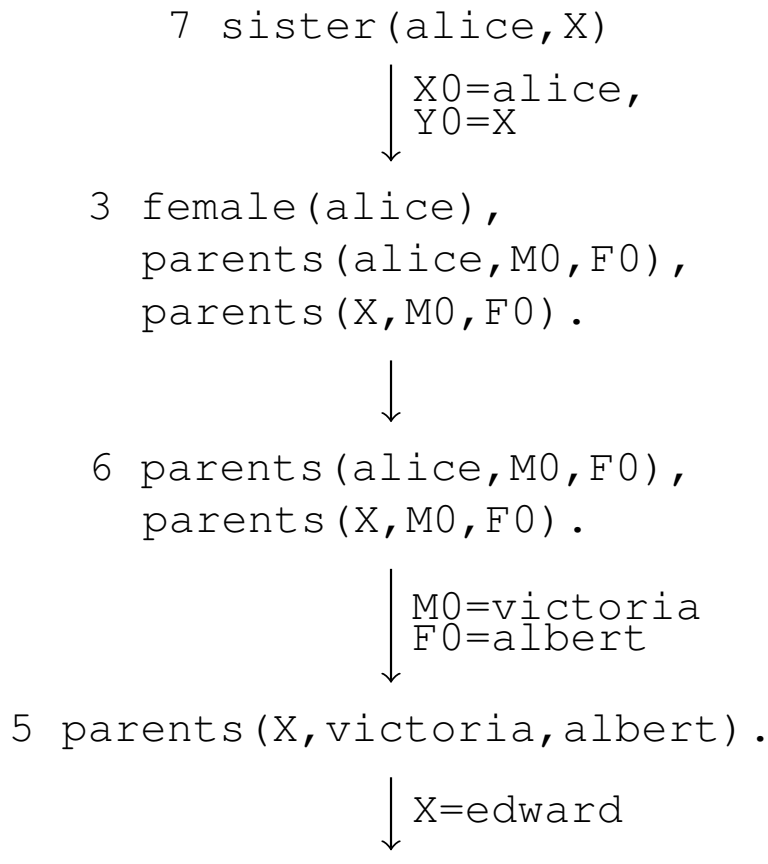
Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```



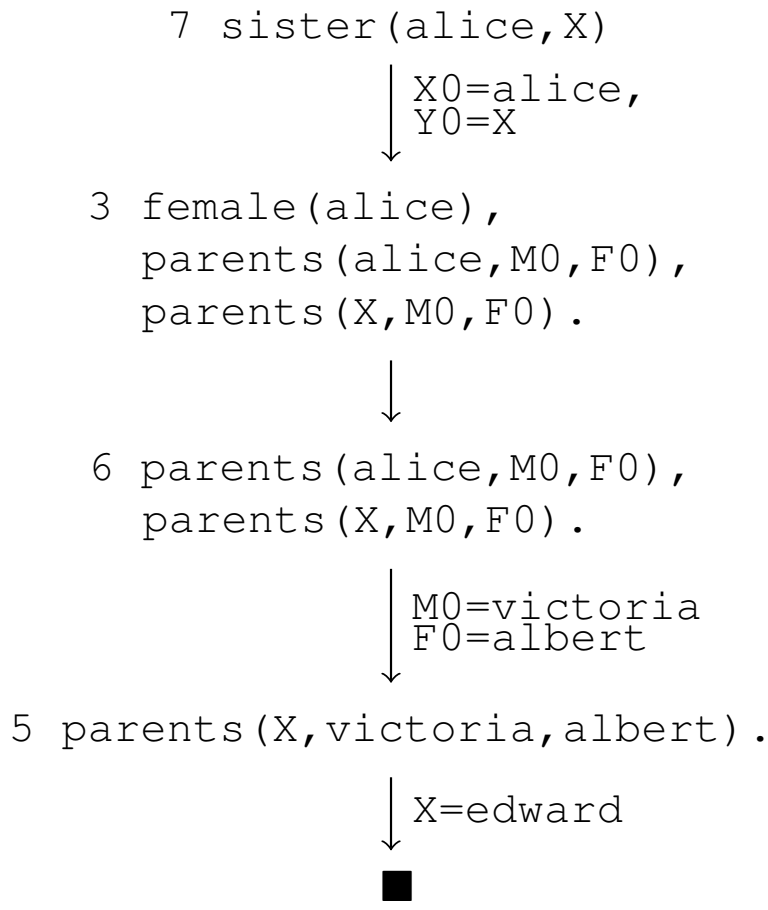
Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```



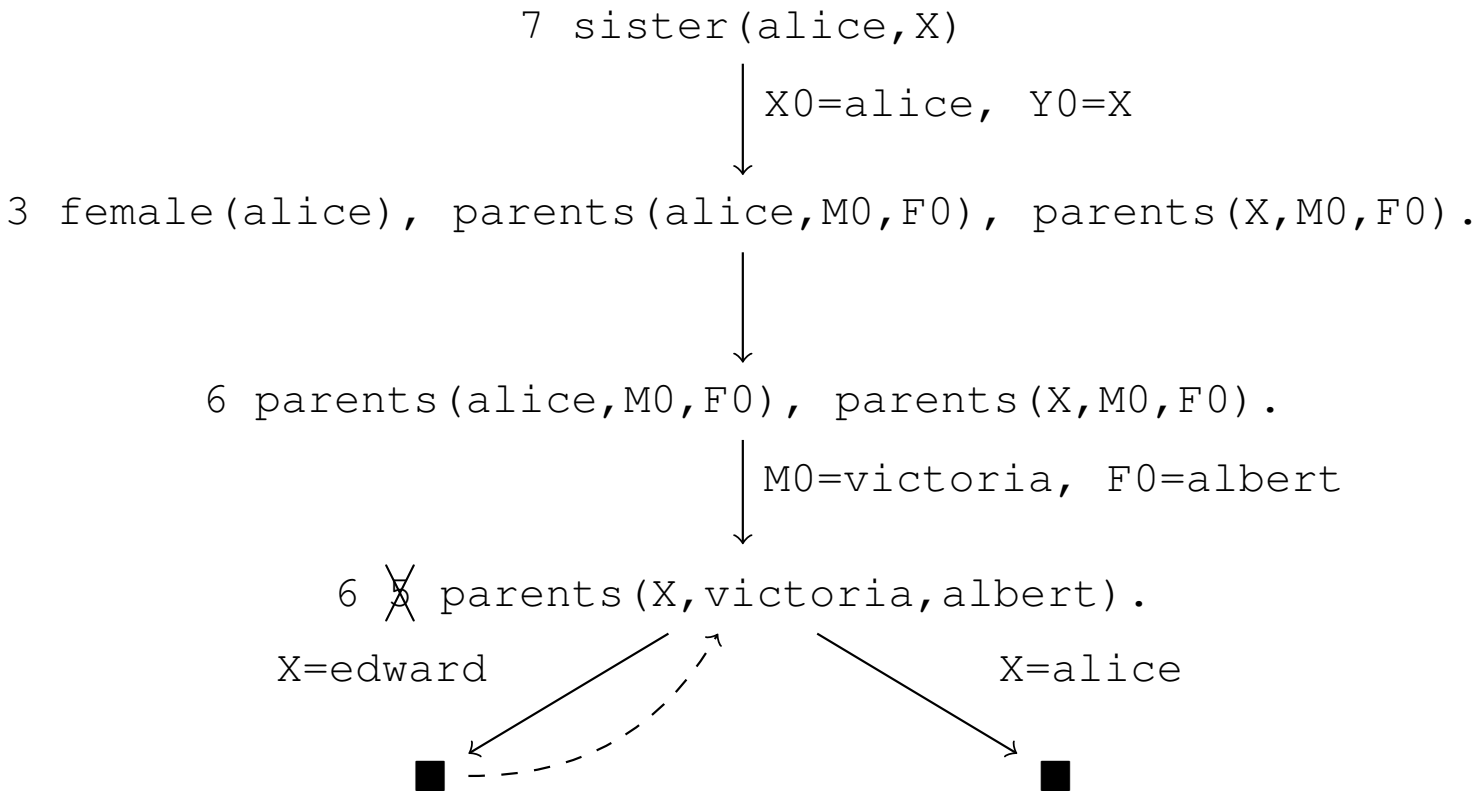
Who's Sister Is Alice?

```
(1) male(albert).
(2) male(edward).
(3) female(alice).
(4) female(victoria).
(5) parents(edward,
      victoria,
      albert).
(6) parents(alice,
      victoria,
      albert).
(7) sister(X, Y):-
      female(X),
      parents(X, M, F),
      parents(Y, M, F).
```



Answer: X = edward.

Complete Derivation Tree



Answer: X = edward.

Answer: X = alice.