Name: .... 28 Jan 2020

Matrikelnummer: ....

Studienkennzahl: ....

Final Exam / Klausur
Computer Algebra (326.105)
(no books / ohne Unterlagen)

Please note:

- *Written documents or electronic tools are NOT allowed in this final exam.*
- *Fill in your name and student ID in the problem sheet — before you start working. On top of every page of your solution write your name.*
- *Hand in the problem sheet together with your solutions. You will find it on the web page of the lecture.*
- *Start a new page for every problem. Hand in your solution ordered w.r.t. the numbers of the problems.*

You may give answers either in English or in German.
Man kann auf Englisch oder Deutsch antworten.

Explain your answers. Simply giving the result or "yes/no" is not enough.
Antworten sind zu begründen. Nur das Ergebnis oder "ja/nein" genügt nicht.

(1) Consider polynomials in $\mathbb{Q}[x, y]$. Let $F = \{f_1, f_2, f_3\}$, and $I = \langle F \rangle$, with

$$f_1 = 2y^2 - x + 1,\ f_2 = xy + 2y,\ f_3 = x^2 + x - 2\ .$$

(a) Is F a Gröbner basis for I w.r.t. the lexicographic ordering with $x < y$?
(b) Which of the polynomials $g = 3xy^2 - 2y^2 + x^3 - x^2, h = xy + 2$ is in I?

(2) Consider polynomials in $\mathbb{Q}[x, y]$.
Prove: *If G is a reduced Gröbner basis w.r.t. the lexicographic ordering with $x < y$, then G can contain at most 1 polynomial purely in x.*

(3) Explain: What is the Chinese Remainder Problem in $\mathbb{Z}$, and how can it be solved?

(4) We consider the problem of computing the greatest common divisor of polynomials in $\mathbb{Z}[x]$.
What are the main subproblems, and how can they be solved?

(5) Suppose $a(x), b(x), p(x), q(x)$ are non-constant polynomials in $\mathbb{Q}[x]$. Furthermore, suppose $\deg(p) < \deg(b)$, $\deg(q) < \deg(a)$, and $p(x)a(x) + q(x)b(x) = 0$.
Does this imply that a and b have a non-constant common factor?