to be prepared for 10.12.2019

Exercise 1. Show that the result of applying the Euclidean Algorithm in $K[x]$ to any pair of polynomials f, g is a reduced Gröbner basis for $\langle f, g \rangle$.

Exercise 2. Compute the normed reduced Gröbner basis w.r.t. the lexicographic ordering with $x < y$ for the ideal generated by

$$\begin{aligned} f_1 &= xy^2 + x^2 + x \\ f_2 &= x^2y + x \end{aligned}$$

in $\mathbb{Z}_3[x, y]$.

Exercise 3. Given are the polynomials in $\mathbb{Q}[x, y, z, t]$

$$\begin{aligned} f_1 &= xyzt + x^2y - z \\ f_2 &= x^3y - xy + z^4t \\ f_3 &= x^2 + y^2 - z^2 - t^2 \end{aligned}$$

Compute a Gröbner basis of the ideal $\langle f_1, f_2, f_3 \rangle$.

Exercise 4. Use Gröbner bases for solving over $\mathbb{C}$:

$$\begin{aligned} f_1(x, y, z) &= xz - xy^2 - 4x^2 - \frac{1}{4} = 0, \\ f_2(x, y, z) &= y^2z + 2x + \frac{1}{2} = 0, \\ f_3(x, y, z) &= x^2z + y^2 + \frac{1}{2}x = 0. \end{aligned}$$

Exercise 5. Consider the polynomials

$$\begin{aligned} f_1(x, y) &= x^2y + xy + 1, \\ f_2(x, y) &= y^2 + x + y \end{aligned}$$

in $\mathbb{Z}_3[x, y]$. Compute a Gröbner basis for the ideal $\langle f_1, f_2 \rangle$ w.r.t. the graduated lexicographical ordering with $x < y$. Show intermediate results.