to be prepared for 07.01.2020

Exercise 1. Consider the linear system

$$\begin{aligned} a-b-f+h-j+k &= 0 \\ -c+d-f+h-i+l &= 0 \\ e+f-g-h+i+j-k-l &= 0. \end{aligned}$$

Decide whether these equations imply that

$$-a+b-e+g-i+l=0.$$

Exercise 2. Consider the ideals I and J in $\mathbb{R}[x,y,z]$.

$$\begin{aligned} I &= \langle xs^2+xt^2+x-2s,\ ys^2+yt^2+y-2t,\ zs^2+zt^2+z-s^2-t^2+1\rangle \cap \mathbb{R}[x,y,z] \\ J &= \langle x^2+y^2+z^2-1\rangle \end{aligned}$$

Determine whether they are equal.

Exercise 3. Consider the following system of algebraic equations

$$\begin{aligned} x^2+y^2+z^2-t^2 &= 0 \\ x^2-y^2+z^2-t^2 &= 0 \\ y^3-6y^2+12y-6 &= 0. \end{aligned}$$

Decide whether it has a solution.

Exercise 4. Consider the ideal $I \subseteq \mathbb{R}[x,y]$ generated by the polynomials

$$x^2+y^2-1, \qquad x^2-y^2-1.$$

Decide whether I coincides with its radical.

Exercise 5. Use Gröbner basis techniques for computing an implicite representation of the parametrized surface

$$x=\frac{2s}{s^2+t^2+1}, \quad y=\frac{2t}{s^2+t^2+1}, \quad z=\frac{2st}{s^2+t^2+1}.$$