

Logic 1, WS 2013. Homework 2, given Oct 20, due Nov 7

1. Define the meta-function $\text{Vars}[\varphi]$ which gives set of propositional variables of the propositional formula φ .

Hint: use the induction principle suggested by the definition of propositional logic formulae.

2. Write the tables of the boolean functions corresponding to $\neg, \vee, \Rightarrow, \Leftrightarrow$.

3. Prove that for any propositional formulae $\varphi_1, \varphi_2, \dots, \varphi_n, \psi$, if $(\varphi_1 \wedge \varphi_2 \wedge \dots \wedge \varphi_n) \Rightarrow \psi$ is valid, then $\varphi_1, \varphi_2, \dots, \varphi_n \models \psi$.

(See the style used in the lecture for proving the opposite implication.)

4. Prove $P, \neg Q \Rightarrow \neg P \models Q$ by constructing the truth table of the associated implication.

5. Define the syntax and the semantics of disjunction over sets and find the truth value of the empty disjunction. (In the lecture the same study was performed for conjunction.)