Logic 1, WS 2012. Homework 4, given Nov 15, due Nov 22.

1. Evaluate the truth of the formula $(P[a] \wedge (\underset{x}{\forall}(P[x] \Rightarrow P[f[x]]))) \Rightarrow P[f[f[a]]]$ under the interpretation I:
$D = \{0, 1, 2\}$;
$a_I = 1$;
$f_I[0] = 2,\ f_I[1] = 2,\ f_I[2] = 0$;
$P_I[0] = \mathbb{F},\ P_I[1] = \mathbb{T},\ P_I[2] = \mathbb{F}$.

2. Find counterexamples φ, ψ which disprove the false equivalences:

$$(\underset{x}{\exists}\varphi) \wedge (\underset{x}{\exists}\psi) \ \equiv\ \underset{x}{\exists}(\varphi \wedge \psi),$$

$$(\underset{x}{\forall}\varphi) \vee (\underset{x}{\forall}\psi) \ \equiv\ \underset{x}{\forall}(\varphi \vee \psi).$$

3. Prove the following equivalence by reducing both sides to CNF:

$$(\underset{x}{\forall}P[x]) \Rightarrow Q \ \equiv\ \underset{x}{\exists}(P[x] \Rightarrow Q).$$

4. Prove that if the formula $\underset{x}{\exists}P[x]$ is satisfiable, then the formula $P[a]$ is also satisfiable.

5. Prove that if the formula $\underset{x}{\forall}P[x, f[x]]$ is satisfiable, then the formula $\underset{x}{\forall}\underset{y}{\exists}P[x, y]$ is also satisfiable.