Exercises discussed on January 22, 2013

46. Execute the algorithm HYPER step by step (with assistance of a computer algebra system) to determine all hypergeomtric solutions of the recurrence

$$3(3n+5)y(n) - (9n^2+27n+17)y(n+1) + (n+2)(3n+2)y(n+2) = 0.$$

47. Use the program Hyper.m to determine the solutions of the following recurrences:

$$2\left(3n^2+7n+6\right)a_n + (n+2)\left(3n^2+n+2\right)a_{n+2} - \left(3n^3+16n^2+15n+10\right)a_{n+1} = 0,$$

with $a_0 = 0, a_1 = -2$;

$$\begin{aligned} 0 = &\left(9n^2+25n+17\right)b_n - (n+1)\left(81n^4+324n^3+437n^2+226n+37\right)b_{n+1} \\ &+ (n+1)(n+2)(2n+3)(3n+4)(3n+5)\left(9n^2+7n+1\right)b_{n+2}, \end{aligned}$$

with $b_0 = -1, b_1 = -2$;

$$\begin{aligned} 0 =&(n-1)n^2 c_{n+3} - (n-1)\left(n^3+6n^2+4n+1\right)c_{n+2} \\ &+ \left(3n^3+6n^2-3n-2\right)(n+1)c_{n+1} - 2n(n+1)^3 c_n, \end{aligned}$$

with $c_0 = 0, c_1 = 1, c_2 = 2$.

48. Design an algorithm which takes as input two rational functions $c_0(x), c_1(x) \in \mathbb{K}(x)$ and a hypergeometric sequence $(a_n)_{n\geq 0}$ and which decides whether the equation

$$c_1(n)s_{n+1} + c_0(n)s_n = a_n$$

has a hypergeometric solution $(s_n)_{n\geq 0}$. Assume for simplicity that $c_1(n) \neq 0 \neq c_0(n)$ for all $n \in \mathbb{N}$. (Hint: One way to handle this problem is to think of a substitution that turns the equation into a telescoping equation and then Gosper's algorithm is applicable.)