

Exercises discussed on January 15, 2013

43. Compute a hypergeometric closed form of the sum $s_n = \sum_{k=0}^n \frac{4}{(2k-1)(2k+1)}$ by applying Gosper's algorithm.
44. Use the implementation of Gosper's algorithm in the package `zb.m` available at

<http://www.risc.jku.at/research/combinat/software/PauleSchorn/index.php>

to determine a closed form of the following sums, if they exist:

(a) $s_n = \sum_{k=0}^n (k-1)k!2^{-k}$

(b) $s_n = \sum_{k=0}^n (k+1)^2 k!$

(c) $s_n(a) = \sum_{k=0}^n a^k$

(d) $s_n(a) = \sum_{k=0}^n \binom{a}{k}$

45. Given polynomials $u(x), v(x), t(x)$ determine a degree bound for the polynomial solution $y(x)$ of the Gosper equation

$$u(x)y(x+1) - v(x)y(x) = t(x)$$

analogously to the discussion of the algorithm POLY (see also "A=B"), i.e., distinguish the three cases:

- $\deg u(x) \neq \deg v(x)$ OR $\text{lcv}(x) \neq \text{lcv}(x)$
- $\deg u(x) = \deg v(x)$ AND $\text{lcv}(x) = \text{lcv}(x)$ and either the terms of second highest degree cancel, or they do not cancel.