Exercises discussed on December 4, 2012

35. Let $C(x) = \sum_{n\geq 0} C_n x^n$ be the generating function of the Catalan numbers. In the lecture we deduced from the equation $xC(x)^2 - C(x) + 1 = 0$ the explicit representation $C(x) = \frac{1-\sqrt{1-4x}}{2x}$. What rules out the possibility $C(x) = \frac{1+\sqrt{1-4x}}{2x}$?

36. Prove or disprove that the multiplicative inverse of $C(x)$ (the generating function of the Catalan numbers) is holonomic.

37. Let $a(x) \in \mathbb{C}[\![x]\!]$ be an algebraic power series and suppose that $b(x) \in \mathbb{C}[\![x]\!]$, $b(0) = 0$, is such that $a(b(x)) = x$. Show that $b(x)$ is algebraic.

38. Let $(a_n)_{n\geq 0}$ be the coefficient sequence of an algebraic power series. Show that then the sequence of partial sums $\left(\sum_{k=0}^{n} a_k\right)_{n\geq 0}$ is the coefficient sequence of an algebraic power series.