

## Exercises discussed on November 20, 2012

27. Show that the sequence of Harmonic numbers  $(H_n)_{n \geq 0}$  is not C-finite.

28. Use the package `GeneratingFunctions`<sup>1</sup> to

- guess a recurrence for the sequence  $(F_{3n})_{n \geq 0}$  (you can use the Mathematica built-in function `Fibonacci[n]` to generate the data);
- guess a recurrence for the sequence  $(L_{2n})_{n \geq 0}$  (you can use the Mathematica built-in function `LucasL[n]` to generate the data);
- compute the recurrences for  $a_n = F_{3n} + L_{2n}$  and  $b_n = F_{3n}L_{2n}$  using closure properties.

29. Determine the hypergeometric function representation of

$$(a) \quad \frac{1}{x} \log(1+x) = \sum_{n \geq 0} \frac{(-1)^n}{n+1} x^n$$

$$(b) \quad \cos(x) = \sum_{n \geq 0} \frac{(-1)^n}{(2n)!} x^{2n}$$

30. Show that in  $\mathbb{Q}[[x]]$  the hypergeometric function  $y(x) = {}_2F_1 \left( \begin{matrix} a & b \\ c \end{matrix}; x \right)$  satisfies the differential equation:

$$x(1-x)y''(x) + (c - (a+b+1)x)y'(x) - aby(x) = 0.$$

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<sup>1</sup>available at <http://www.risc.jku.at/research/combinat/software>