Exercises discussed on October 30, 2012

15. Show the exponential law for formal power series:

$$\exp(ax)\exp(bx) = \exp((a+b)x), \qquad a, b \in \mathbb{K}.$$

16. Let $(a_n(x))_{n\geq 0}, (b_n(x))_{n\geq 0}$ be convergent sequences of formal power series with respective limits $a(x), b(x) \in \mathbb{K}[\![x]\!]$.

    Show that then also $(c_n(x))_{n\geq 0}$ with $c_n(x) = a_n(x) + b_n(x)$ is a convergent sequence of formal power series with limit $a(x) + b(x)$.

17. Prove the reflection formula, i.e., show that for x an indeterminate and $k \in \mathbb{N}$:

$$\binom{x}{k} = (-1)^k \binom{k-x-1}{k}.$$

18. Give a direct proof of the binomial theorem in the ring of formal power series, i.e., show that for $f(x) \in \mathbb{C}[\![x]\!]$ with $f(0) = 0$ and $\lambda \in \mathbb{C}$ that

$$(1+f(x))^\lambda := \sum_{n\geq 0} \binom{\lambda}{n} f(x)^n \in \mathbb{C}[\![x]\!].$$