Exercises discussed on January 24, 2012

54. Execute the algorithm HYPER step by step (with assistance of a computer algebra system) to determine all hypergeomtric solutions of the recurrence

$$3(3n+5)y(n) - (9n^2+27n+17)y(n+1) + (n+2)(3n+2)y(n+2) = 0.$$

55. Execute Zeilberger's algorithm step by step (with assistance of a computer algebra system) to determine the hypergeometric closed form of

$$s(n) = \sum_{k=0}^{n} \frac{2^k}{k!(n-k)!}.$$

56. Use the program Hyper.m to determine the solutions of the following recurrences:

$$2\left(3n^2+7n+6\right)a_n + (n+2)\left(3n^2+n+2\right)a_{n+2} - \left(3n^3+16n^2+15n+10\right)a_{n+1} = 0,$$

with $a_0 = 0, a_1 = -2$;

$$\begin{aligned} 0 = &\left(9n^2+25n+17\right)b_n - (n+1)\left(81n^4+324n^3+437n^2+226n+37\right)b_{n+1} \\ &+ (n+1)(n+2)(2n+3)(3n+4)(3n+5)\left(9n^2+7n+1\right)b_{n+2}, \end{aligned}$$

with $b_0 = -1, b_1 = -2$;

$$\begin{aligned} 0 = &(n-1)n^2 c_{n+3} - (n-1)\left(n^3+6n^2+4n+1\right)c_{n+2} \\ &+ \left(3n^3+6n^2-3n-2\right)(n+1)c_{n+1} - 2n(n+1)^3 c_n, \end{aligned}$$

with $c_0 = 0, c_1 = 1, c_2 = 2$.

57. Use the program zb.m to determine recurrences for the following sums:

   (a) $s(n) = \sum_{k=0}^{n} \binom{\lambda}{k}\binom{\mu}{n-k}$ for λ, μ formal parameters.

   (b) $s(n) = \sum_{k=0}^{n} \binom{n+k}{2k}(-4)^{-k}$.

   (c) $s(n) = \sum_{k=0}^{n} \binom{n+2k}{2k}\binom{2k}{k}\frac{(-1)^k}{k+1}$.

   (d) $s(n) = \sum_{k=0}^{n} \binom{n}{k}^2\binom{n+k}{k}^2$.

   Where possible, determine closed form solutions (e.g., using Hyper).