Exercises discussed on January 17, 2012

49. Compute a hypergeometric closed form of the sum $s_n = \sum_{k=0}^{n} \frac{4}{(2k-1)(2k+1)}$ using the Guess and Prove strategy.

50. Compute a hypergeometric closed form of the sum $s_n = \sum_{k=0}^{n} \frac{4}{(2k-1)(2k+1)}$ by applying Gosper's algorithm.

51. Show that harmonic numbers do not admit a closed form as hypergeometric term plus constant using Gosper's algorithm.

52. Use the implementation of Gosper's algorithm in the package zb.m avalaible at

    http://www.risc.jku.at/research/combinat/software/PauleSchorn/index.php

    to determine a closed form of the following sums, if they exist:

    (a) $s_n = \sum_{k=0}^{n} (k-1)k!2^{-k}$
    (b) $s_n = \sum_{k=0}^{n} (k+1)^2 k!$
    (c) $s_n(a) = \sum_{k=0}^{n} a^k$
    (d) $s_n(a) = \sum_{k=0}^{n} \binom{a}{k}$
    (e) $s_n = \sum_{k=0}^{n} \frac{4k-1}{(2k-1)^2} \binom{2k}{k}^2 4^{-2k}$
    (f) $s_n = \sum_{k=0}^{n} \frac{4k-1}{2k-1} \binom{2k}{k} 4^{-2k}$

53. Given polynomials $u(x), v(x), t(x)$ determine a degree bound for the polynomial solution $y(x)$ of the Gosper equation

    $$u(x)y(x+1) - v(x)y(x) = t(x)$$

    analogously to the discussion of the algorithm POLY (see also "A=B"), i.e., distinguish the three cases:

    – $\deg u(x) \neq \deg v(x)$ OR $\text{lc}\,u(x) \neq \text{lc}\,v(x)$
    – $\deg u(x) = \deg v(x)$ AND $\text{lc}\,u(x) = \text{lc}\,v(x)$ and either the terms of second highest degree cancel, or they do not cancel.