Exercises discussed on January 10, 2012

45. Implement a program in your favourite computer algebra system that sums a given polynomial sequence using

    (a) falling factorial representation.

    (b) interpolation (you may use built-in commands to execute the interpolation, e.g., in Mathematica the command InterpolatingPolynomial).

    Compute some test cases, in particular compare the timings for the sparse and dense polynomial given in testcases.m.

46. Let $\big(a_n\big)_{n\geq 0}$ be a C-finite sequence satisfying the recurrence with characteristic polynomial

    $$\chi(x) = c_0 + c_1 x + \cdots + c_{r-1}x^{r-1} + x^r,$$

    with $\chi(1) = 0$, more precisely, $\chi(x) = (x-1)^m \bar{\chi}(x)$, $m \geq 1, \bar{\chi}(1) \neq 0$. Let $\bar{q}(x)$ be such that $\bar{\chi}(x) = (x-1)\bar{q}(x) + \bar{\chi}(1)$ and define $\big(b_n\big)_{n\geq 0} := \bar{q}(x) \bullet \big(a_n\big)_{n\geq 0}$.
    Show that

    $$b_{n+1} - b_n + \bar{\chi}(1)a_n = \bar{p}(n), \qquad n \geq 0,$$

    for some $\bar{p} \in \mathbb{K}[x]$ with $\deg\big(\bar{p}(x)\big) \leq m-1$. (I.e., fill in the missing details from the lecture.)

47. Express $s_n = \sum_{k=0}^{n} a_k$ in terms of $a_n, a_{n+1}, \ldots$, where the sequence $\big(a_n\big)_{n\geq 0}$ is given by the recurrence

    $$a_{n+4} - a_{n+3} - 3a_{n+2} + 5a_{n+1} - 2a_n = 0, \qquad a_0 = 3, a_1 = -4, a_2 = 9, a_3 - 12.$$

48. Express $s_n = \sum_{k=0}^{n} a_k$ in terms of $a_n, a_{n+1}, \ldots$, where the sequence $\big(a_n\big)_{n\geq 0}$ is given by the recurrence

    $$a_{n+2} + a_{n+1} - 6a_n = 0, \qquad a_0 = 1, a_1 = -1.$$

Enjoy the vacations and Happy New Year!