Exercises discussed on November 15, 2011

22. Let the signless Stirling numbers of the first kind $C(n,k)$ denote the number of permutations of $\{1,2,\ldots,n\}$ with exactly k cycles. Derive a recurrence relation for $C(n,k)$ and starting from this recurrence, derive a recurrence relation for the Stirling numbers of the first kind $S_1(n,k) := (-1)^{n-k}C(n,k)$.

23. Let x be an indeterminate and $n \in \mathbb{N}$. Show that

    (a) $x^n = \sum_{k=0}^{n} S_2(n,k)x^{\underline{k}}$

    (b) $x^{\underline{n}} = \sum_{k=0}^{n} S_1(n,k)x^k$

24. (Tower of Hanoi) Given a tower of n disks initially stacked in increasing order on one of three pegs, the task is to transfer the entire tower to one of the other pegs, moving only one disk at a time and never moving a larger disk onto a smaller one. Let a_n denote the minimal number of moves needed.

    Find a recurrence for a_n. Compute the first few values and guess a closed form solution. Derive the closed form solution using techniques from the lecture.

25. Given n people numbered from 1 to n sitting at a round table. Starting from person 1 in clockwise order every second person leaves until only one person remains (the first person to leave is person 2). Let $J(n)$ denote the number of the remaining person. Determine $J(n)$.

26. Let the sequence $\big(f(n)\big)_{n\geq 0}$ be recursively defined by

    $$f(n+3) + 2f(n+2) - f(n+1) - 2f(n) = 0, \qquad f(0) = 1,\ f(1) = -1,\ f(2) = 2.$$

    What is the companion matrix of this recurrence? Determine the solution of the recurrence.