```
Clear[x]

Solve[1 - x + x^5 == 0, x]
```

```
{{x → Root[1 - #1 + #1^5 &, 1]}, {x → Root[1 - #1 + #1^5 &, 2]},
 {x → Root[1 - #1 + #1^5 &, 3]}, {x → Root[1 - #1 + #1^5 &, 4]}, {x → Root[1 - #1 + #1^5 &, 5]}}
```

`Root[`f`, `k`]`
represents the exact k^{th} root of the polynomial equation $f[x] == 0$.

```
Solve[1 - x + x^5 == 0, x] // N
```

```
{{x → -1.1673}, {x → -0.181232 - 1.08395 i}, {x → -0.181232 + 1.08395 i},
 {x → 0.764884 - 0.352472 i}, {x → 0.764884 + 0.352472 i}}
```

```
Root[1 - x + x^5, 1] // N
```

```
-1.1673
```

`ToNumberField[`a`, `θ`]`	express the algebraic number a in the number field generated by θ
`ToNumberField[{`a_1`, `a_2`, …}, `θ`]`	express the a_i in the field generated by θ
`ToNumberField[{`a_1`, `a_2`, …}]`	express the a_i in a common extension field generated by a single algebraic number

```
ToNumberField[{√2, √3}]
```

$$\left\{\text{AlgebraicNumber}\left[\text{Root}\left[1-10\,\#1^2+\#1^4\,\&,\,4\right],\,\left\{0,\,-\frac{9}{2},\,0,\,\frac{1}{2}\right\}\right],\;\text{AlgebraicNumber}\left[\text{Root}\left[1-10\,\#1^2+\#1^4\,\&,\,4\right],\,\left\{0,\,\frac{11}{2},\,0,\,-\frac{1}{2}\right\}\right]\right\}$$

`AlgebraicNumber[`θ`, {`c_0`, `c_1`, …, `c_n`}]` represent the algebraic number $c_0 + c_1\,\theta + \ldots + c_n\,\theta^n$ in $\mathbb{Q}[\theta]$

```
ra = Root[1 - 10 #1^2 + #1^4 &, 4] // N;
√2 == -9/2 ra + 1/2 ra^3
```

```
True
```

`AlgebraicNumber` automatically makes the generator of the extension an algebraic integer and the coefficient list equal in length to the degree of the extension.

```
A = AlgebraicNumber[Root[2 #^4 - 3 # + 2 &, 1], {1, 2, 3, 4, 5, 6}]
```

$$\text{AlgebraicNumber}\left[\text{Root}\left[16-12\,\#1+\#1^4\,\&,\,1\right],\,\left\{-4,\,\frac{7}{4},\,3,\,\frac{1}{2}\right\}\right]$$

```
A // N
```

```
-4.78802 + 12.3388 i
```

```
r = Root[2 #^4 - 3 # + 2 &, 1];

F[x_] = 1 + 2 x + 3 x^2 + 4 x^3 + 5 x^4 + 6 x^5;

F[r] // N
```

```
-4.78802 + 12.3388 i
```

```
b = AlgebraicNumber[Root[16 - 12 #1 + #1^4 &, 1], {-4, 7/4, 3, 1/2}] // N
```

```
-4.78802 + 12.3388 ⅈ
```

Das Beispiel von vorher:

```
ToNumberField[{√2, √3}]
```

```
{AlgebraicNumber[Root[1 - 10 #1^2 + #1^4 &, 4], {0, -9/2, 0, 1/2}],
 AlgebraicNumber[Root[1 - 10 #1^2 + #1^4 &, 4], {0, 11/2, 0, -1/2}]}
```

```
ToNumberField[{√2, √3}, Root[1 - 10 #1^2 + #1^4 &, 1]]
```

```
{AlgebraicNumber[Root[1 - 10 #1^2 + #1^4 &, 1], {0, 9/2, 0, -1/2}],
 AlgebraicNumber[Root[1 - 10 #1^2 + #1^4 &, 1], {0, -11/2, 0, 1/2}]}
```

```
z = 2 √2;
y = 3 √3;
```

```
z / 2 + y / 3 + 1 / (z / 2 - y / 3) // N
```

```
-1.77636 × 10^-15
```

```
ToNumberField[√2 + √3 + 1/(√2 - √3)]
```

```
0
```

`RootReduce` transforms `AlgebraicNumber` objects to `Root` objects.

```
A
```

```
AlgebraicNumber[Root[16 - 12 #1 + #1^4 &, 1], {-4, 7/4, 3, 1/2}]
```

```
RootReduce[A]
```

```
Root[220525 - 20389 #1 + 2288 #1^2 - 32 #1^3 + 16 #1^4 &, 2]
```

Arithmetic within a fixed finite extension of rationals is much faster than arithmetic within the field of all complex algebraic numbers.

```
Clear[x, y, z]
```

```
{a, b, c} = {ⅈ, √2, Root[#^3 - 2 # + 3 &, 1]};
f = (-2 y z (7 + x - y + z^2) + (6 + x^2 + 2 y) (-11 + x y + z^2)) / (2 y z (-4 - x + 3 y z) - (6 + x^2 + 2 y) (2 - 2 x + z^3));
```

```
RootReduce[h = f /. {x → a, y → b, z → c}] // Timing
```

{0.187, Root[127 463 137 729 603 858 692 + 15 069 520 316 552 576 640 #1 + 3 151 085 417 830 482 145 156 #1^2 − 10 938 243 534 840 099 267 928 #1^3 + 14 492 589 303 525 156 688 533 #1^4 − 7 171 605 298 335 082 808 820 #1^5 − 947 445 370 794 828 405 814 #1^6 + 2 510 661 531 113 587 622 448 #1^7 − 606 316 032 776 880 635 517 #1^8 − 100 899 537 810 316 084 288 #1^9 + 74 049 398 920 051 042 942 #1^10 − 12 985 018 306 589 245 140 #1^11 + 879 298 673 075 259 913 #1^12 &, 4]}

```
h // N
```

−0.0523076 + 0.171974 i

```
({aa, bb, cc} = ToNumberField[{a, b, c}]) // Timing
```

$$\left(d = \frac{-2\,y\,z\,(7 + x - y + z^2) + (6 + x^2 + 2\,y)\,(-11 + x\,y + z^2)}{2\,y\,z\,(-4 - x + 3\,y\,z) - (6 + x^2 + 2\,y)\,(2 - 2\,x + z^3)}\ /.\ \{x \to aa,\ y \to bb,\ z \to cc\}\right)\ //\ \text{Timing}$$

```
d // N
```

−0.0523076 + 0.171974 i

MinimalPolynomial[*a*]
give a pure function representation of the minimal polynomial over the integers of the algebraic number *a*
MinimalPolynomial[*a*, *x*]
give the minimal polynomial of the algebraic number *a* as a polynomial in *x*

$$\text{MinimalPolynomial}\left[\sqrt{2} + \sqrt{3}\right]$$

1 − 10 #1^2 + #1^4 &

$$\text{MinimalPolynomial}\left[\sqrt{2} + \sqrt{3},\ x\right]$$

$1 - 10\,x^2 + x^4$

$$\text{AlgebraicIntegerQ}\left[\frac{1}{2}\left(1 + \sqrt{5}\right)\right]$$

True

$$\text{MinimalPolynomial}\left[\frac{1}{2}\left(1 + \sqrt{5}\right)\right]$$

−1 − #1 + #1^2 &

$$\text{AlgebraicIntegerQ}\left[\frac{1}{4}\left(1 + \sqrt{5}\right)\right]$$

False

$$\text{MinimalPolynomial}\left[\frac{1}{4}\left(1 + \sqrt{5}\right)\right]$$

−1 − 2 #1 + 4 #1^2 &