

Logic 1, WS 2009. Homework 7, given Jan 14, due Jan 21.

1. Prove by sequent calculus:

$$((\forall x P[x]) \Rightarrow Q) \Rightarrow (\exists x (P[x] \Rightarrow Q)).$$

2. Prove by sequent calculus:

$$(\forall x (P[x] \Rightarrow Q)) \Rightarrow ((\exists x P[x]) \Rightarrow Q).$$

3. Find an unsatisfiable set of ground instances of the clauses:

$$P[x, a, g[x, b]], P[f[y], z, g[f[a], b]]$$

4. Prove by refutation and resolution that

$$\forall x P[x, a]$$

is a logical consequence of

$$\forall x ((\exists y P[x, y]) \Rightarrow P[x, a])$$

and

$$\neg \exists x \forall y \neg P[x, y].$$