

**Logic 1, WS 2008. Homework 1, given Oct 15, due Oct 22**

1. Define inductively the function “the set of propositional variables of a propositional formula”.
2. Define inductively the function “the depth of a propositional formula”. The depth of a formula is the depth of the tree which represents the formula. This tree has logical connectives as internal nodes and propositional variables as the leaves. Every subtree corresponds to a subformula, has as root the main logical connective of this subformula, and as subtrees the sub-subformulae related by this logical connective. Examples:  $\text{Depth}[A] = 0$ ;  $\text{Depth}[A \vee B] = 1$ ;  $\text{Depth}[A \Rightarrow (A \vee B)] = 2$ .  $\text{Depth}[(A \wedge B) \Rightarrow (A \vee B)] = 2$ .
3. Prove that the depth of a formula is less or equal to the length of that formula.
4. Write the truth table for the formula:

$$((A \vee B) \wedge (A \Rightarrow C) \wedge (B \Rightarrow C)) \Rightarrow C.$$

5. Using rewriting to normal form, prove the equivalence of the formulae:  $(A \vee B) \Rightarrow C$  and  $(A \Rightarrow C) \wedge (B \Rightarrow C)$ .