Final Exam

Computeralgebra (326017)

WS 2009/2010 — 26.1.2010

Matr.Nr./SKZ	(1)	(2)	(3)	(4)	(5)	(opt)	Sum
0756463/201	12	12	6	–	12	–	42
0655389/201	4	12	4	–	8	0	28
0755078/201	12	6	6	6	12	0	42
0756825/201	12	4	6	12	12	12	58
0755682/201	5	6	6	–	12	0	29
0756502/201	3	11	–	0	2	–	16
0755422/201	4	6	6	11	10	6	43
0857738/091	12	8	6	10	–	–	36
0956945/403	11	8	6	9	12	–	46
0657345/201	2	4	6	–	10	0	22
0756186/201	12	12	6	–	6	12	48
0456096/201	–	–	–	–	–	–	0
0957171/786	12	8	6	12	–	12	50
0655481/201	–	12	6	12	0	6	36
0755409/201	12	6	6	12	12	4	52
9456332/201	12	6	3	–	3	–	24
0857339/786	12	6	6	3	4	–	31

Solutions:

(1) (a) $\text{res}_y = 2(x+2)(x-1)(x-6)$
(b) $\text{lc}(a(-2,y)) = 0 = \text{lc}(b(-2,y))$, so not extendable
(c) leading coeff.s don't vanish at $x=1$, $(x,y)=(1,1)$ is common sol.

(2) system of (1) has 2 solutions: $(1,1),(6,-1/4)$; so there must be 2 irreducible power products w.r.t. the Gröbner basis; so no new polynomial can be added to the basis $\{a,b,c,d\}$; actually $\{c,d\}$ is reduced GB

(3) (a) $h \longrightarrow^*_G \neq 0$, so $h \notin I$; $h \longrightarrow^*_H \neq 0$, so $h \notin J$
(b) for different orderings $>$ the normed reduced Gröbner bases can be different; but $\forall i: g_i \longrightarrow^*_H 0$, and $\forall i: h_i \longrightarrow^*_G 0$; so $I = J$

(4) Lemma 8.2.5 in Winkler, Polynomial Algorithms in Computer Algebra

(5) in $\mathbb{Z}[x]$: $\gcd(f,g) = x-1$, $r := \text{res}(f/(x-1), f/(x-1)) = 9$;
modulo 2: $\gcd(f_{(2)}, g_{(2)}) = x+1$, $2 \nmid r$;
modulo 3: $\gcd(f_{(3)}, g_{(3)}) = x^3+2$, $3 | r$;

(6) for instance $R = \mathbb{Q}[x_i | i \in \mathbb{N}]$;
$\langle x_1 \rangle \subset \langle x_1, x_2 \rangle \subset \cdots R$ is an infinite strictly increasing chain of ideals