Thinking, Speaking, Writing

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Can you do it also under the assumption that your neighbor does not speak English/German? or is blind?

Explanations

Most students in math/cs are well trained in explaining math/cs to poeple who already know (mostly, the teacher). The challenge is to explain to people who do not yet know!

Exercise for 2 persons: A explains B the concept of dimension for vectorspaces. B should pretend to know the concept of vectorspace, but not the concept of dimension, and interrupt with questions whenever something is not clear.

Frequent Shortcomings

Small errors are allowed and can be corrected tacitly by the listener, but some errors lead necessarily to misunderstandings (unless the listener knows the topic anyway). These errors need to be corrected explicitly.

Gaps or usage of terminology that is not explained and not known to the listener.

Oral definition of functions. One sometimes meets students that just never thought about how to say this properly. Do it once, if not done already.

Exercises for Defining Functions

Exercise 1: Define without writing the function $\mathbb{R} \longrightarrow \mathbb{R}$, $x \mapsto x^2$.

Exercise 2: Define without writing the closure operator on subsets of a topological space.

Exercise 2': For an error-correcting code correction capability 1, define the correction function.

Exercise 3: For any vectorspace, there exists a natural function from it to its double dual. Define it without writing.

Exercise 3': Define a function for the time complexity of a program.

Questions

If possible, a question should be precise. Sometimes it is necessary to explain to the speaker what it was that I did not understand. Before answering a question, I should make sure that he/she understood the question.

Many questions can be answered by repeating the already said one more time!

Performance

The most important is to have something to explain. This gives your talk a goal, and creates a dramatic situation. (The goal may fail!)

One may or may not use non-mathematical/non-cs elements to make the talk more interesting: jokes, stories, eye contact, involvement of persons from the audience.

There are mathematical elements of language that may make your talk more interesting: surprises, riddles.

Written Math/CS

Text in math/cs is often read outside the intended context, and also often with less assumptions on the reader (mostly assumed to a mathematician/computer scientist). However, we can also distinguish many situations here: refereed paper, lecture notes, software manual, code documentation.

Most text in math/cs is carefully constructed. The care is necessary to facilitate the reader to understand the written explanations.

Recommended articles in Wikipedia: Moufang loop, hyperbolic geometry, root system.

About Style

Individual persons have different opinions about long/short sentences, word repetions, long/short paragraphs, indentations etc.

Exercise

You are giving a course in linear algebra at the JKU to first year students. For the lectore notes write a section on dimension.

Proofs

Proofs can be done in mind, in speaking (explanation) or in writing. As a rule of thumb, a written proof is a text that helps the reader to do the proof by himself/herself in his/her mind.

A Proof Situation

I have some good reason to believe that a certain mathematical statement is true. But I am not sure. In order to be sure, I try to prove it.

Exercise: For any two relations R and S on a set G, we define the composite RS as the relation

$$x\ RS\ y :\Longleftrightarrow \exists z(x\ R\ z \wedge z\ S\ y).$$

I am telling you now: if R and S are transitiv, and $RS = SR$, then RS is transitive.

One Last Word on Proofs

Finding a proof and writing it down may be hard construction work. Once you are close to finished, try to make it also beautiful. And once it is done, sit back a while and enjoy the sparkling shine of your proof!

Spoiler for the Last Exercise

Assume $a\ RS\ b$ and $b\ RSc$. Then $\exists d$ such that $a\ R\ d$ and $d\ S\ b$, and $\exists e$ such that $b\ R\ e$ and $e\ S\ c$. Hence $d\ SR\ e$. But $SR = RS$, so $\exists f$ such that $d\ R\ f$ and $f\ S\ e$. From $a\ R\ d$ and $d\ R\ f$ we get $a\ R\ f$ by transitivity. From $f\ S\ e$ and $e\ S\ c$ we get $f\ S\ c$ by transitivity. Hence $a\ RS\ c$.