Logic 1, WS 2007. Homework 7, given Dec 13, due Jan 10.

1. In the usual theory of natural numbers, find a contradiction introduced by the definition:

$$\forall_{x,y} P[x] \Leftrightarrow (x + y = 0).$$

2. In the theory of real numbers, consider the predicate P defined by:

$$\forall_{x,y} P[x, y] \Leftrightarrow \exists_t (x = \sin t \wedge y = \cos t).$$

Study the possibility of defining a function f by:

$$\forall_{x,y} f[x] = y \Leftrightarrow P[x, y],$$

or an alternative formula in which x and/or y must obey some condition or/and the equivalence is replaced by an implication.

3. Reformulate the verification condition

$$\forall_{x:I_F:\neg Q[x]} \forall_y (O_F[R[x], y] \Rightarrow I_C[x, y])$$

such that $R[x]$ is not used, but the specification of R.

4. Reformulate the verification condition

$$\forall_{x:I_F:\neg Q[x]} \forall_y (O_F[R[x], y] \Rightarrow O_F[x, C[x, y]])$$

such that $R[x]$ and $C[x, y]$ are not used, but the specifications of R and C.