

**Logic 1, WS 2006. Exam exercises May 22, 2007**

NAME:

MATRIKELNR.:

1. Give a natural style proof of the formula:

$$((P \vee Q) \Rightarrow R) \Rightarrow ((P \Rightarrow R) \wedge (Q \Rightarrow R)).$$

2. Construct the sequent proof-tree of the formula:

$$((Q \vee R) \wedge (Q \Rightarrow P) \wedge (R \Rightarrow S)) \Rightarrow (P \vee S).$$

3. Show how to compute the truth value of the formula  $\exists_x P[x, f[a]]$  under the interpretation  $I$  given by:

$$D = \{1, 2\}$$

$$a_I = 2$$

$$f_I[1] = 2, f_I[2] = 1$$

$$P_I[1, 1] = P_I[2, 2] = \mathbb{F}, P_I[1, 2] = P_I[2, 1] = \mathbb{T}$$

4. Use resolution in order to infer the formula  $R$  from the clauses:

$$P \vee Q \vee R, \quad \overline{P} \vee R, \quad \overline{Q} \vee R.$$

5. By equivalent transformations, obtain the existentially quantified formula which is equivalent to  $(\forall_x Q[x]) \Rightarrow P$ .

6. In the following normalized formula,  $x, y, z$  are variables and  $a$  is constant. Prove by resolution that the formula is not satisfiable.

$$\{\neg P[x] \vee Q[f[a], x], P[f[z]], \neg Q[y, y]\}.$$

7. Construct the sequent proof of the formula:

$$(\forall_x (P[x] \Rightarrow Q)) \Rightarrow ((\forall_x \neg P[x]) \vee Q)$$

Note: Subjects 2, 6, and 7 get 15 points, the others get 10.