

*Information Systems*  
*Relational Databases*

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# Outline

The Relational Model (Continues from the Previous Lecture)

Integrity Constraints

Functional Dependencies

# Outline

The Relational Model (Continues from the Previous Lecture)

Integrity Constraints

Functional Dependencies

# Integrity Constraints

- ▶ Integrity Constraint: A boolean expression that is associated with some database and is required to evaluate at all times to TRUE.

# Integrity Constraints. Examples

Suppliers-and-parts database satisfy the constraints:

- ▶ Every supplier status value is in the range 1 to 100 inclusive.
- ▶ Every supplier in London has status 20.
- ▶ If there are any parts at all, at least one of them is blue.
- ▶ No two distinct suppliers have the same supplier number.
- ▶ etc.

All examples today from suppliers-and-parts database.

# Integrity Constraints

- ▶ Constraints must be formally declared to the DBMS and DBMS must enforce them.
- ▶ Declaring constraints is a matter of using relevant features of the database language.
- ▶ Enforcing them is a matter of the DBMS monitoring updates that might violate the constraints and rejecting those that do.

## Example

To enforce the constraint *Every supplier status value is in the range 1 to 100 inclusive*, the DBMS will have to monitor all operations that attempt

- ▶ to insert a new supplier, or
- ▶ change an existing supplier's status.

# Integrity Constraints

- ▶ Overall “shape” of integrity constraints:  
IF  
a certain tuple appears in certain relvars,  
THEN  
that tuple satisfies a certain condition.

# Classification of Constraints

- ▶ Type constraint: A definition of the set of values that constitute a given type.
- ▶ Attribute constraint: Constrains values a given attribute is permitted to assume.
- ▶ Relvar constraint: Constrains values a given relvar is permitted to assume.
- ▶ Database constraint: Constrains values a given database is permitted to assume.

# Type Constraints

- ▶ Definition of the set of values that constitute a given type.
- ▶ Example:  
TYPE WEIGHT POSSREP { D DECIMAL (5,1) }  
CONSTRAINT D > 0.0 AND D < 5000.0
- ▶ Meaning:  
Legal values of type WEIGHT are precisely those
  - ▶ that can possibly be represented by decimal numbers of five digits precision with one digit after the decimal point,
  - ▶ where the decimal number in question is greater than zero and less than 5000.
- ▶ Type constraints are thought of being checked during the execution of some selector invocation.
- ▶ WEIGHT ( 7500.0 ) will raise an exception at run time (value out of range).

# Attribute Constraints

- ▶ Declaration to the effect that a specified attribute of a specified relvar is of a specified type.

- ▶ Example:

```
VAR S BASE RELATION
{ S#      S#
  SNAME  NAME
  STATUS  INTEGER
  CITY   CHAR } ...;
```

- ▶ Part of the attribute definition itself, can be identified by the corresponding attribute name.

# Relvar Constraints

- ▶ Constrain possible values of a given relvar.
- ▶ Example:
  - ▶ Every supplier status value is in the range 1 to 100 inclusive.
- ▶ For all supplier numbers  $s\#$ , all names  $sn$ , all integers  $st$  and all character strings  $sc$ :
  - ▶ IF a tuple with  $S\#$   $s\#$ ,  $SNAME$   $sn$ ,  $STATUS$   $st$ , and  $CITY$   $sc$  appears in the suppliers relvar  $S$ ,
  - ▶ THEN  $st$  is greater than or equal to 1 and less than or equal to 100.
- ▶ Constraint for  $S$ .

# Relvar Constraints

- ▶ Any given relvar can be subject to many constraints.
- ▶ Example:
  - ▶ Every supplier status value is in the range 1 to 100 inclusive.
  - ▶ No two distinct suppliers have the same supplier number.
- ▶ **The** relvar constraint: Conjunction of all constraints for the relvar.
- ▶ Golden Rule:
  - ▶ No update operation must ever assign to any relvar R a value that causes the constraint for R to evaluate to FALSE.

# Database Constraints

- ▶ Database constraint: Conjunction of all the relvar constraints for all relvars contained in the database.
- ▶ Golden Rule:
  - ▶ No update operation must ever assign to any database a value that causes even the database constraint to evaluate to FALSE.

# Integrity and Views

- ▶ Constrained relvars can be both base relvars and views.
- ▶ If a view  $R_V$  is derived from a base relvar  $R_B$ , then a constraint for  $R_V$  can be derived from the corresponding constraint for  $R_B$  just at  $R_V$  is derived from  $R_B$ .

# Integrity and Views

## Example

Let SST be a view obtained by projecting S over S#, SNAME, and STATUS:

S	S#	SNAME	ST	CITY
	S1	Smith	20	London
	S2	Jones	10	Paris
	S3	Blake	30	Paris
	S4	Clark	20	London
	S5	Adams	30	Athens

SST	S#	SNAME	ST
	S1	Smith	20
	S2	Jones	10
	S3	Blake	30
	S4	Clark	20
	S5	Adams	30

- ▶ Constraint: Every supplier status value is in the range 1 to 100 inclusive.

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SST	S#	SNAME	ST
	S1	Smith	20
	S2	Jones	10
	S3	Blake	30
	S4	Clark	20
	S5	Adams	30

- ▶ Constraint: Every supplier status value is in the range 1 to 100 inclusive.
- ▶ For S: For all supplier numbers  $s\#$ , all names  $sn$ , all integers  $st$  and all character strings  $sc$ :
  - ▶ IF a tuple with S#  $s\#$ , SNAME  $sn$ , STATUS  $st$ , and CITY  $sc$  appears in the relvar S, THEN  $1 \leq st \leq 100$ .

# Integrity and Views

## Example

Let SST be a view obtained by projecting S over S#, SNAME, and STATUS:

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	S4	Clark	20
	S5	Adams	30

- ▶ Constraint: Every supplier status value is in the range 1 to 100 inclusive.
- ▶ For SST: For all supplier numbers  $s\#$ , all names  $sn$ , and all integers  $st$ :
  - ▶ IF a tuple with S#  $s\#$ , SNAME  $sn$ , STATUS  $st$ , appears in the relvar SST, THEN  $st \ 1 \leq st \leq 100$ .

# Keys

## Candidate Key

- ▶ Let  $K$  be a set of attributes of relvar  $R$ . Then  $K$  is a **candidate key** for  $R$  iff it has both of the following properties:
  - ▶ Uniqueness: No legal value of  $R$  ever contains two distinct tuples with the same value for  $K$ .
  - ▶ Irreducibility: No proper subset of  $K$  has the uniqueness property.
- ▶ Every relvar has at least one candidate key.
- ▶ candidate keys do not include any attributes that are irrelevant for unique identification purposes.

# Keys

## Example

Examples of Candidate Keys.

- ▶ VAR S BASE RELATION  
{ S# S#  
SNAME NAME  
STATUS INTEGER  
CITY CHAR }  
KEY { S# }

Simple candidate key.

- ▶ VAR SP BASE RELATION  
{ S# S#  
P# P#  
QTY QTY }  
KEY { S#, P# }

Composite candidate key.

# Keys

## Example

Examples of Candidate Keys.

- ▶ Several candidate keys are possible

VAR MARRIAGE BASE RELATION

```
{ HUSBAND  NAME  
  WIFE      NAME  
  DATE      DATE }
```

KEY { HUSBAND, DATE }

KEY { DATE, WIFE }

KEY { WIFE, HUSBAND }

# Keys

- ▶ A candidate key definition is a shorthand for a certain relvar constraint.

## Example

- ▶ { S# } is a candidate key.
- ▶ Corresponding constraint: No two distinct suppliers have the same supply number.
- ▶ A bit more formally:  
For all supplier numbers  $x\#$  and  $y\#$ , all names  $xn$  and  $yn$ , all integers  $xt$  and  $yt$ , and all character strings  $xc$  and  $yc$ :
  - ▶ IF tuples with  
S#  $x\#$ , SNAME  $xn$ , STATUS  $xt$ , CITY  $xc$  and  
S#  $y\#$ , SNAME  $yn$ , STATUS  $yt$ , CITY  $yc$   
appear in the suppliers relvar S,
  - ▶ THEN IF  $x\# = y\#$   
THEN  $xn = yn$ ,  $xt = yt$ , and  $xc = yc$ .

# Keys

- ▶ A given relvar can have two or more candidate keys.
- ▶ Exactly one of those keys (at least for base relvars) are chosen as the **primary key**.
- ▶ The others are called **alternate keys**.

# Keys

- ▶ A **foreign key** in a relvar  $R_2$  is a set of attributes of  $R_2$ , say  $FK$ , such that:
  - ▶ There exists a relvar  $R_1$  ( $R_1$  and  $R_2$  not necessarily distinct) with a candidate key  $CK$ .
  - ▶ Each value of  $FK$  (or a renamed copy of  $FK$ ) in the current value of  $R_2$  is identical to the value of  $CK$  in some tuple in the current value of  $R_1$ .

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- ▶ Points:
  - ▶ Every value of  $FK$  must appear as a value of  $CK$ , the converse is not necessary.

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  - ▶ Every value of  $FK$  must appear as a value of  $CK$ , the converse is not necessary.
  - ▶  $FK$  is simple or composite according as  $CK$  is simple or composite.

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  - ▶ An  $FK$  value represents a reference to the tuple containing the matching  $CK$  value (the referenced tuple).

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  - ▶ The constraint that values of  $FK$  must match the values of  $CK$  is known as **referential constraint**.

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  - ▶ Each value of  $FK$  (or a renamed copy of  $FK$ ) in the current value of  $R_2$  is identical to the value of  $CK$  in some tuple in the current value of  $R_1$ .
- ▶ Points:
  - ▶ Every value of  $FK$  must appear as a value of  $CK$ , the converse is not necessary.
  - ▶  $FK$  is simple or composite according as  $CK$  is simple or composite.
  - ▶ An  $FK$  value represents a reference to the tuple containing the matching  $CK$  value (the referenced tuple).
  - ▶ The constraint that values of  $FK$  must match the values of  $CK$  is known as **referential constraint**.
  - ▶  $R_2$  is the referencing relvar and  $R_1$  is a referenced relvar.

# Keys

- ▶ Referential constraints in the suppliers-and-parts database can be represented by means of the **referential diagram**:

$$S \leftarrow_{S\#} SP \rightarrow_{P\#} P.$$

- ▶ A given relvar can be both referenced and referencing:

$$R_n \rightarrow R_{n-1} \rightarrow \cdots \rightarrow R_1.$$

Referential chain from  $R_n$  to  $R_1$ .

- ▶ A relvar might include a foreign key whose values are required to match the values of some candidate key in the same relvar (self-referencing):

```
VAR EMP BASE RELATION
```

```
  { EMP# EMP#, ... MGR_EMP# MGR_EMP#, ... }
```

```
  KEY { EMP# }
```

```
  FOREIGN KEY { RENAME MGR_EMP# AS EMP# }  
                REFERENCES EMP
```

- ▶ Referencing cycles are allowed.

# Keys

- ▶ The relational model includes the following rule:  
**Referential Integrity**: The database must not contain any unmatched foreign key values.

# Keys

- ▶ A foreign key definition is a shorthand for a certain relvar constraint:

## Example

- ▶ { S# } is a foreign key for shipments.
- ▶ Constraint: Every shipment involves an existing supplier (supplier-and-parts db).
- ▶ A bit more formally:  
For all supplier number  $s\#$ , all part number  $p\#$ , and all integers  $q$ :
  - ▶ IF a tuple with S#  $s\#$ , P#  $p\#$ , QTY  $q$  appears in the shipments relvar SP,
  - ▶ THEN there exists a name  $sn$ , an integer  $st$ , and a character string  $sc$  such that the tuple with S#  $s\#$ , SNAME  $sn$ , STATUS  $st$ , CITY  $sc$  appears in the suppliers relvar S.

# Referential Actions

## Example

- ▶ `DELETE S WHERE S# = S# ('S1') ;`
- ▶ Deletes supplier tuple for S1.
- ▶ Assume:
  - ▶ The database includes some shipments for S1
  - ▶ The application does not delete those shipments.
- ▶ Then the system raises an exception when it checks the referential constraint from shipments to suppliers.

Alternative approach possible.

# Referential Actions

## Example

### Alternative Approach

- ▶ `DELETE S WHERE S# = S# ('S1') ;`
- ▶ Deletes supplier tuple for S1.
- ▶ Alternative approach: If the database includes some shipments for S1, delete those shipments as well.

- ▶ The effect achieved by extending the foreign key definition:

```
VAR SP BASE RELATION { ... } ...  
    FOREIGN KEY { S# } REFERENCES S  
        ON DELETE CASCADE ;
```

- ▶ `ON DELETE CASCADE` defined a delete rule for the foreign key.
- ▶ `CASCADE`: referential action. The `DELETE` operation on `S` will cascade to delete matching tuples in the shipments relvar as well.

# Referential Actions

- ▶ RESTRICT: Another common referential action.
- ▶ In case of DELETE (suppliers-and-parts db), RESTRICT would mean that the operation is restricted to the case when there are no matching shipments.
- ▶ Otherwise, the operations are rejected.
- ▶ Omitting a referential action for a particular foreign key is equivalent to specifying to NO ACTION.
- ▶ Referential actions make sense also for UPDATE.

# Triggers

- ▶ A trigger is a statement (in the query language) the DBMS executes automatically whenever a set of conditions becomes true.

## Example

- ▶ Let LONDON\_SUPPLIER be a view:  
CREATE VIEW LONDON\_SUPPLIER  
AS SELECT S#, SNAME, STATUS  
FROM S  
WHERE CITY = 'London' ;
- ▶ When trying to insert a row in this view, a row will be inserted in the underlying base table S with the default value for CITY.
- ▶ If the default value is not 'London', the row will not appear in the view.

# Triggers

## Example (Cont.)

- ▶ Create a triggered procedure:  
CREATE TRIGGER LONDON\_SUPPLIER\_INSERT  
INSTEAD OF INSERT ON LONDON\_SUPPLIER  
REFERENCING NEW ROW AS R  
FOR EACH ROW  
INSERT INTO S ( S#, SNAME, STATUS, CITY )  
VALUES ( R.S#, R.SNAME, R.STATUS, 'London' );

# Triggers

## Example (Cont.)

- ▶ Create a triggered procedure:  
CREATE TRIGGER LONDON\_SUPPLIER\_INSERT  
INSTEAD OF INSERT ON LONDON\_SUPPLIER  
REFERENCING NEW ROW AS R  
FOR EACH ROW  
INSERT INTO S ( S#, SNAME, STATUS, CITY )  
VALUES ( R.S#, R.SNAME, R.STATUS, 'London' );
- ▶ Inserting a row in the view will cause a row to be inserted into the underlying base table with CITY value equal to London instead of the default value, and the new row will appear in the view.

# Triggers

## Example (Cont.)

- ▶ Create a triggered procedure:  
CREATE TRIGGER LONDON\_SUPPLIER\_INSERT  
INSTEAD OF INSERT ON LONDON\_SUPPLIER  
REFERENCING NEW ROW AS R  
FOR EACH ROW  
INSERT INTO S ( S#, SNAME, STATUS, CITY )  
VALUES ( R.S#, R.SNAME, R.STATUS, 'London' );
- ▶ CREATE TRIGGER specifies
  - ▶ *The event*: an operation on the database (INSERT ON LONDON\_SUPPLIER).
  - ▶ *The condition*: a boolean expression that has to evaluate to true in order the action to be executed (No condition specified explicitly in the example above);
  - ▶ *The action*: the triggered procedure proper (INSERT INTO S).

# Outline

## The Relational Model (Continues from the Previous Lecture)

Integrity Constraints

Functional Dependencies

# Functional Dependencies

- ▶ Functional Dependence (FD): Many-to-one relationship from one set of attributes to another within a given relvar, satisfying certain conditions.
- ▶ Should distinguish clearly between
  - (a) the value of a given relvar at a given point of time, and
  - (b) the set of possible values that the given relvar might assume at different times.

# Functional Dependencies

## Functional Dependence, Case (a)

- ▶ Let  $r$  be a relation, and  $X$  and  $Y$  be arbitrary subsets of the set of attributes of  $r$ .
- ▶  $Y$  is **functionally dependent** on  $X$ , written  $X \rightarrow Y$  iff whenever two tuples of  $r$  agree on their  $X$  value, they also agree on their  $Y$  value.

# Functional Dependencies

## Example

### Functional Dependence, Case (a)

SPC	S#	CITY	P#	QTY
	S1	London	P1	100
	S1	London	P2	100
	S2	Paris	P1	200
	S2	Paris	P2	200
	S3	Paris	P2	300
	S4	London	P2	400
	S4	London	P4	400
	S4	London	P5	400

FD's:

- $\{S\# \} \rightarrow \{CITY\}$
- $\{S\#, P\# \} \rightarrow \{QTY\}$
- $\{S\#, P\# \} \rightarrow \{CITY\}$
- $\{S\#, P\# \} \rightarrow \{S\# \}$
- $\{S\#, P\# \} \rightarrow \{S\#, P\#, CITY, QTY\}$
- $\{S\# \} \rightarrow \{QTY\}$
- $\{QTY\} \rightarrow \{S\# \}$

# Functional Dependencies

## Functional Dependence, Case (b)

- ▶ Let  $R$  be a relation variable, and  $X$  and  $Y$  be arbitrary subsets of the set of attributes of  $R$ .
- ▶  $Y$  is **functionally dependent** on  $X$ , written  $X \rightarrow Y$  iff in every possible value legal of  $R$ , whenever two tuples agree on their  $X$  value, they also agree on their  $Y$  value.

# Functional Dependencies

## Example

Functional Dependence, Case (b)

SPC	S#	CITY	P#	QTY
	S1	London	P1	100
	S1	London	P2	100
	S2	Paris	P1	200
	S2	Paris	P2	200
	S3	Paris	P2	300
	S4	London	P2	400
	S4	London	P4	400
	S4	London	P5	400

Some of the  
FD's for SCP:

$$\{S\#, P\#\} \rightarrow \{QTY\}$$

$$\{S\#, P\#\} \rightarrow \{CITY\}$$

$$\{S\#, P\#\} \rightarrow \{S\#\}$$

$$\{S\#, P\#\} \rightarrow \{S\#, P\#, \\ CITY, QTY\}$$

---

$$\{S\#\} \not\rightarrow \{QTY\}$$

$$\{QTY\} \not\rightarrow \{S\#\}$$

# Functional Dependencies

- ▶ From now on FD refers to FD, case (b).
- ▶ If  $X$  is a candidate key for relvar  $R$ , then all attributes  $Y$  of  $R$  must be functionally dependent on  $X$ .
- ▶ FD for a given relvar can be large.
- ▶ Problem: Find a smaller (ideally, minimal) subset of FD's for a given relvar that implies all the FD's for that relvar.
- ▶ Why is this problem interesting?
  - ▶ FD's are certain integrity constraints.
  - ▶ DBMS should enforce them.
  - ▶ If we find a smaller subset required in the problem statement, it is sufficient for the DBMS to enforce just FD's from that subset. All the other FD's will be enforced automatically.

# Functional Dependencies

- ▶ Trivial FD: Right side is a subset of the left side (e.g.,  $\{S\#, P\#\} \rightarrow \{P\#\}$ ).
- ▶ Trivial dependencies are not interesting in practice.

# Functional Dependencies

Armstrong's axioms:

- ▶ Reflexivity: If  $B \subseteq A$ , then  $A \rightarrow B$ .
- ▶ Augmentation: If  $A \rightarrow B$ , then  $A \cup C \rightarrow B \cup C$ .
- ▶ Transitivity: If  $A \rightarrow B$  and  $B \rightarrow C$ , then  $A \rightarrow C$ .

Armstrong's axioms are sound and complete.

# Functional Dependencies

Additional rules (follow from Armstrong's axioms):

- ▶ Self-determination: If  $A \rightarrow A$ .
- ▶ Decomposition: If  $A \rightarrow B \cup C$ , then  $A \rightarrow B$  and  $A \rightarrow C$ .
- ▶ Union: If  $A \rightarrow B$  and  $A \rightarrow C$ , then  $A \rightarrow B \cup C$ .
- ▶ Composition: If  $A \rightarrow B$  and  $C \rightarrow D$ , then  $A \cup C \rightarrow B \cup D$ .

# Functional Dependencies

## Example

- ▶ Given: relvar  $R$  with attributes  $A, B, C, D, E, F$  and FD's:  
 $\{A\} \rightarrow \{B, C\}, \{B\} \rightarrow \{E\}, \{C, D\} \rightarrow \{E, F\}.$
- ▶ Show:  $\{A, D\} \rightarrow \{F\}$  holds for  $R$ .
- ▶
  1.  $\{A\} \rightarrow \{B, C\}$  (given)
  2.  $\{A\} \rightarrow \{C\}$  (1, decomposition)
  3.  $\{A, D\} \rightarrow \{C, D\}$  (2, augmentation)
  4.  $\{C, D\} \rightarrow \{E, F\}$  (given)
  5.  $\{A, D\} \rightarrow \{E, F\}$  (3 and 4, transitivity)
  6.  $\{A, D\} \rightarrow \{F\}$  (5, decomposition)

# Functional Dependencies. Computing the Closure

- ▶ Given: a relvar  $R$ , a set of attributes  $Z$  for  $R$ , and a set  $S$  of FD's that hold for  $R$ .
- ▶ Compute: The set of all attributes of  $R$  that are functionally dependent on  $Z$ —the closure  $Z^+$  of  $Z$  under  $S$ .

- ▶ Algorithm:

$CLOSURE[Z, S] := Z$  ;

do “Forever” ;

    for each FD  $X \rightarrow Y$  in  $S$

        do ;

            if  $X \subseteq CLOSURE[Z, S]$

                then  $CLOSURE[Z, S] := CLOSURE[Z, S] \cup Y$ ;

        end

    if  $CLOSURE[Z, S]$  did not change on this iteration

        then leave the loop ;

end

# Functional Dependencies. Computing the Closure

## Example

- ▶ Given: a relvar  $R$  with attributes  $A, B, C, D, E, F$  and FD's:

$$\{A\} \rightarrow \{B, C\}$$

$$\{E\} \rightarrow \{C, F\}$$

$$\{B\} \rightarrow \{E\}$$

$$\{C, D\} \rightarrow \{E, F\}$$

- ▶ Compute: The closure of  $\{A, B\}$ .
- ▶ Computation:
  1. Initialization: The closure is  $\{A, B\}$
  2. After the first iteration of the algorithm:  $\{A, B, C, E\}$
  3. After the second iteration:  $\{A, B, C, E, F\}$
  4. After the third iteration:  $\{A, B, C, E, F\}$
  5. Answer:  $\{A, B, C, E, F\}$ .

# Functional Dependencies

Important corollary:

- ▶ Given: A set  $S$  of FD's.
- ▶ Decide: Does a specific FD  $X \rightarrow Y$  follow from  $S$ ?
- ▶ Decision procedure:
  1. Compute the closure  $X^+$  of  $X$  under  $S$ .
  2. Check whether  $Y$  is a subset of  $X^+$ .
  3. If yes,  $X \rightarrow Y$  follows from  $S$ , and does not, otherwise. (No need to compute  $S^+$ .)

# Functional Dependencies

- ▶ If  $S_1$  and  $S_2$  are two sets of FD's with  $S_1^+ \subseteq S_2^+$ , then  $S_2$  is called a cover of  $S_1$ .
- ▶ If DBMS enforces the FD's in  $S_2$ , then it will automatically be enforcing those in  $S_1$ .
- ▶ If  $S_1^+ = S_2^+$ , then  $S_1$  and  $S_2$  are equivalent.
- ▶ A set  $S$  of FD's is irreducible iff
  1. The right side of every FD in  $S$  is a singleton set.
  2. The left side of each FD in  $S$  is irreducible—no attribute can be discarded from them without changing  $S^+$ .
  3. No FD can be discarded from  $S$  without changing  $S^+$ .
- ▶ Every set of FD's is equivalent to at least one irreducible set.

# Summary

- ▶ The **integrity** problem is the problem to ensure that the data is consistent.
- ▶ Integrity constraints take the general form:
  - IF  
a certain tuple appears in certain relvars,  
THEN  
that tuple satisfies a certain condition.
- ▶ The **relvar constraint (the relvar predicate)** is the conjunction of all constraints for the relvar.
- ▶ The **database constraint (the database predicate)** is the conjunction of all the constraints for all relvars in the database.
- ▶ **The Golden Rule:**
  - ▶ No update operation must ever assign to any relvar R a value that causes the constraint for R to evaluate to FALSE.

# Summary

- ▶ The integrity constraints represent the **meaning** of the data (semantics).
- ▶ Integrity constraints into four categories:
  - ▶ **Type** constraints.
  - ▶ **Attribute** constraints.
  - ▶ **Relvar** constraints.
  - ▶ **Database** constraints.
- ▶ Keys: **candidate, primary, alternate, foreign**.
- ▶ Candidate keys satisfy **uniqueness** and **irreducibility** properties.

# Summary

- ▶ **Referential constraint:** Values of a given foreign key must match the values of the corresponding candidate key.
- ▶ **Referential actions** and **triggers**.

# Summary

- ▶ **Functional dependency:** A many-to-one relationship between two sets of attributes of given relvar, satisfying a certain functionality condition.
- ▶ **Trivial FD:** Right side is a subset of the left side.
- ▶ Given a set  $S$  of FD's, the **closure**  $S^+$  of that set is the set of all FD's implied by the FD's in  $S$ .
- ▶ **Armstrong's axioms** provide a sound and complete rules to compute the closure of a given set of FD's.

# Summary

- ▶ Given a subset  $Z$  of the set of attributes of relvar  $R$  and a set  $S$  of FD's that hold for  $R$ , the **closure**  $Z^+$  of  $Z$  **under**  $S$  is the set of all attributes  $A$  of  $R$  such that the FD  $Z \rightarrow A \in S^+$ .
- ▶ We gave a simple algorithm for computing  $Z^+$  from  $Z$  and  $S$ .
- ▶ Using the algorithm, one can determine whether a given FD  $X \rightarrow Y$  belongs to  $S^+$ : Just check whether  $Y \in X^+$ .
- ▶ Two sets of FD's  $S_1$  and  $S_2$  are equivalent iff they are covers of each other:  $S_1^+ = S_2^+$ .
- ▶ Every set of FD's is equivalent to at least one irreducible set.
- ▶ If  $I$  is an irreducible set equivalent to  $S$ , enforcing the FD's in  $I$  will automatically enforce the FD's.