Logic 1, WS 2004. Homework 5, given Nov 04, due Nov 11

1. Prove that if the formula $P(a)$ is satisfiable, then the formula $\exists_x P(x)$ is also satisfiable.

2. Prove that if the formula $\forall_x \exists_y P(x, y)$ is satisfiable, then the formula $\forall_x P(x, f(x))$ is also satisfiable.

3. Explain how to construct the Herbrand universe for the formula

$$\forall_x (P(f(x), a) \Rightarrow Q(b, g(x))).$$

4. Bring into the Skolem normal form the negation of the formula:

$$((\forall_x (P(x) \Rightarrow (Q(x) \wedge R(x)))) \;\wedge\; (\exists_x (P(x) \wedge S(x)))) \Rightarrow (\exists_x (R(x) \wedge S(x))).$$

5. Use the resolution method in order to infer the empty clause from the clauses of the formula obtained above.