LVA 326333 Special Topics in Symbolic Computation:

Holonomic Differential Equations and Modular Forms

Summer Semester 2022
Monday, 15:30-17:30, seminar room, RISC, Castle of Hagenberg

1st lecture (survey, etc.): Monday, March 7th, 15:30

Linear differential equations with polynomial coefficients (i.e., holonomic DE) for functions defined on the complex plane play an important role in areas like enumerative combinatorics and special functions (e.g., hypergeometric series, Bessel functions, Legendre and other families of orthogonal polynomials, etc.).

The lecture introduces to the basics of the underlying theory: existence of solutions (incl. aspect of monodromy), representations in terms of power series (e.g., Kummer's 24 solutions to Riemann's DE), aspects of singularity analysis, etc.

Instead of aiming at abstract generality, special emphasis is put on staying as concrete as possible; for instance, much of the theoretical material will be restricted to order 2. This aim goes together with a special focus on concrete examples, many of them will be treated by using computer algebra (Mathematica packages).

The ultimate goal of the lecture is to lead up to recent algorithmic developments which connect modular forms and functions with holonomic functions; the latter being functions satisfying holonomic DEs. Applications concern classical topics like Ramanujan's formulas for the fast computation of pi, and irrationality proofs for zeta(3).

Requirements: solid knowledge of linear algebra and analysis; basic knowledge of complex analysis ("Funktionentheorie") would be helpful. Nevertheless, relevant notions will be explained in detail. - The course should be accessible to beginning Master's students in Computer Mathematics and for students interested in topics as described.

Literature: In addition to research articles, books like "Second Order Differential Equations - Special Functions and Their Classification" (Gerhard Kristensson, Springer) or the classical book "Elements of the Theory of Elliptic Functions" by Naum Ilyich Akhiezer.