Questions on Tropcial Polynomials

Task 1 Plot the tropical polynomial functions defined by

a) $-2 \odot x^3 \oplus -1 \odot x^2 \oplus 1 \odot x \oplus 5$;

b) $x \oplus y \oplus 0$.

Task 2 Let $p : \mathbb{R}^n \to \mathbb{R}$ be a function that is continuous, concave and linear with finitely many components having integer coefficients. Show that p can be represented by a tropical polynomial in $x_1, \ldots, x_n$.

Task 3 Formulate the Fundamental Theorem for tropical polynomial functions and prove it.

Let us emphasize that the Fundamental Theorem cannot hold for tropical polynomials itself. For example

$$x^2 \oplus 17 \cdot x \oplus 2 = x^2 \oplus 1 \cdot x \oplus 2 = (x \oplus 1)^2$$

as function but not as polynomial. The uniqueness of the factorization does not hold anymore in the multivariate case:

$$(x \oplus 0) \odot (y \oplus 0) \odot (x \odot y \oplus 0) = (x \odot y \oplus x \oplus 0) \odot (x \odot y \oplus y \oplus 0)$$