

Tropical Linear Algebra

Task 1 Let us consider the equation

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \otimes \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}.$$

Compute its smallest subsolution and check whether it is also a solution.

Task 2 Let $A = \begin{pmatrix} 1 & \infty \\ \infty & 2 \end{pmatrix}$.

- Compute two distinct eigenvalues of A .
- Compute $\text{trop det}(A)$.

Task 3 Let $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

- Check that -1 is the unique eigenvalue of A .
- Compute the eigenspace of A .

Task 4 Let us consider the matrix

$$\mathfrak{A} = \begin{pmatrix} x & 1 & x \\ 1 & x & -x^2 \\ x & x^2 & x \end{pmatrix} \in \mathbb{C}\langle\langle x \rangle\rangle^{3 \times 3}.$$

- Compute eigenvectors corresponding to the eigenvalues $t, t \pm \sqrt{1 + t^2 - t^4}$.
- Compute the characteristic polynomial of $A = \text{trop}(\mathfrak{A})$.
- Compute the eigenvalue of A and compare to the valuation of the eigenvalues of \mathfrak{A} .

Task 5 Let us consider, for $A = \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}, b = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$, the equation

$$x = A \otimes x \oplus b.$$

Compute its unique solution.