Convex Geometry

Task 1 Let $P \subset \mathbb{R}^3$ be the polyhedron defined by $A \cdot x \leq b$ with

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

a) Draw P and compute different faces.

b) Which of the following properties holds for P?
cone / rational / lineality space is trivial / $\{P\}$ is a polyhedron complex / dimension = 3

Task 2 Let $Y = \text{trop}(\mathbb{V}(x + y + z + 1)) \in \mathbb{C}^3$.

a) Compute $\text{trop}(Y)$, and a polyhedral complex Σ with support equals $\text{trop}(Y)$.

b) Show that Σ is balanced if we set weight one on each maximal dimensional cone.

Task 3 Let Σ be the pure one-dimensional polyhedral fan with cones $\text{pos}((1, 0)) = \mathbb{R}_{\geq 0} \cdot (1, 0)$, $\text{pos}((0, 1)), \text{pos}((-1, 1)), \text{pos}((-1, -1))$. Find all weights for which Σ is balanced.

Task 4 Let $F = x^2y^2 + x^3 + y^3 + 1 \in \mathbb{C}[x, y]$.

a) Express $\text{trop}(\mathbb{V}(F))$ as fan.

b) Compare to the Newton polygon $\mathcal{N}(F)$.

c) What do you obtain when we consider $F \in \mathbb{C}\langle\langle x\rangle\rangle[y]$ with the non-trivial valuation on the field of Puiseux series?

Task 5 Let $F = xy^2 + y + 2z - yz + x \in \mathbb{C}\langle\langle x\rangle\rangle[y, z]$.

a) Compute $\text{trop}(\mathbb{V}(F))$ and compare to $\mathcal{N}(F)$.

b) Compute for different cones the stars with respect to Σ, the fan obtained from $\text{trop}(\mathbb{V}(F))$.