21. Let $a_n = F_n L_n$, where $(F_n)_{n\geq 0}$ denotes the sequence of Fibonacci numbers and $(L_n)_{n\geq 0}$ the sequence of Lucas numbers. Derive a C-finite recurrence that is satisfied by $(a_n)_{n\geq 0}$.

22. Show that the sequence of Harmonic numbers $(H_n)_{n\geq 0}$ is not C-finite.

23. Determine

    (a) the hypergeometric series representations of

    $$\tfrac{1}{x}\log(1+x) = \sum_{n\geq 0} \frac{(-1)^n}{n+1} x^n \qquad \text{and} \qquad \cos(x) = \sum_{n\geq 0} \frac{(-1)^n}{(2n)!} x^{2n}.$$

    (b) the asymptotics of

    $$\frac{3^n}{4n+1}\binom{3n}{n+1}^2 \binom{6n}{2n}^{-1}.$$

24. (a) Characterize all sequences that are both C-finite and hypergeometric.

    (b) Can $f(x) = \sin(x) + \cos(x)$ be expressed as a hypergeometric series?