Algorithmic Combinatorics
Exercises discussed on June 3, 2019

46. Given is the following P-finite recurrence

$$(4n+1)g_{n+2} + 2(4n-1)g_{n+1} - 3(4n+5)g_n = 0, \quad g_0 = 1,\ g_1 = 0.$$

Determine all candidate pairs for $u(n), v(n+2)$ in the algorithm Hyper presented in the lecture and at least one hypergeometric solution to this recurrence.

47. Chebyshev polynomials of the first kind can be defined using the recurrence

$$T_{n+2}(x) - 2xT_{n+1}(x) + T_n(x) = 0, \quad T_0(x) = 1,\ T_1(x) = x.$$

Compute the generating function $F(z) = \sum_{n\geq 0} T_n(x)z^n$.

48. Given are two sequences $(a_n)_{n\geq 0}$ and $(b_n)_{n\geq 0}$. The sequence a_n is hypergeometric satisfying $c_1(n)a_{n+1} = c_0(n)a_n$ for some polynomials c_0, c_1 and b_n is P-finite satisfying

$$d_r(n)b_{n+r} + \cdots + d_1(n)b_{n+1} + d_0(n)b_n = 0,$$

for polynomials d_k. Give a direct proof that the sequence $c_n = a_nb_n$ is P-finite of order r by explicitely computing the recurrence coefficients.