Algorithmic Combinatorics

Exercises discussed on May 20, 2019

40. Implement a program in your favourite computer algebra system that sums a given polynomial sequence using

    (a) falling factorial representation.

    (b) interpolation (you may use built-in commands to execute the interpolation, e.g., in Mathematica the command InterpolatingPolynomial).

    Compute some test cases, in particular compare the timings for the sparse and dense polynomial given in testcases.m.

41. Let T_n be the number of tilings of a $3 \times n$ rectangle with straight trominoes (i.e., 1×3 and 3×1 pieces).

    (a) Determine a recurrence relation for T_n.

    (b) Express the partial sum $s_n = \sum_{k=0}^{n} T_k$ in terms of T_n.

42. Express $s_n = \sum_{k=0}^{n} a_k$ in terms of $a_n, a_{n+1}, \ldots$, where the sequence $\left(a_n\right)_{n \geq 0}$ is given by the recurrence

$$a_{n+3} = 5a_{n+1} - 4a_n, \qquad a_0 = a_1 = 1, \quad a_2 = 2.$$