Algorithmic Combinatorics
Exercises discussed on May 6, 2019

30. Determine the asymptotics of

$$\frac{3^n}{4n+1}\binom{3n}{n+1}^2\binom{6n}{2n}^{-1}.$$

31. Can $f(x) = \sin(x) + \cos(x)$ be expressed as a hypergeometric series?

32. Show that a sequence $\big(a_n\big)_{n\geq 0}$ is holonomic if and only if there exist polynomials $p_0, \ldots, p_r \in \mathbb{K}[x]$ and $q \in \mathbb{K}[x]$ such that

$$p_r(n)a_{n+r} + \cdots + p_1(n)a_{n+1} + p_0(n)a_n = q(n), \qquad n \in \mathbb{N}.$$

33. Show that if $\big(a_n\big)_{n\geq 0}$ is holonomic, then $s_n = \sum_{k=0}^{n} a_k$ is holonomic.

34. Use Mallinger's package GeneratingFunctions to

    (a) compute the defining differential equation for $y(x) = \frac{x}{\sqrt{1-4x}}$ starting from the algebraic equation

    $$(1-4x)y(x)^2 - x^2 = 0;$$

    (b) derive a recurrence relation for the coefficients a_n of $y(x) = \sum_{n\geq 0} a_n x^n$ starting from the differential equation computed in (a).

    Solve the recurrence computed in (b) using your favourite computer algebra system.